

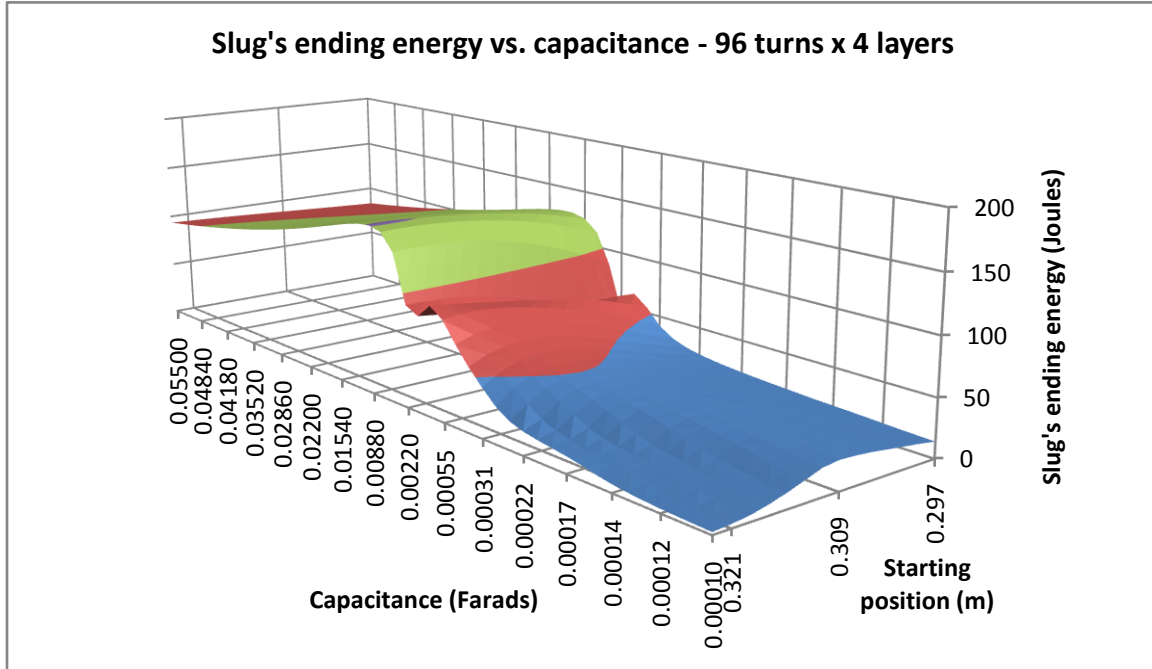
Physics of an axial coil gun

Part IV – Designing a better current waveform

One of the problems identified in Part III of this paper is that the current rises to its peak too early during a run. We need to find some way to “slow down” the current so that more of it flows during the time the slug is passing through the face of the coil. There are two easy modifications which accomplish this result: increasing the capacitance of the power source and / or increasing the inductance of the coil (or, perhaps, adding additional inductance in series with the coil of the gun).

The power supply which was used in all of the runs up to this point is a 100 μF capacitor initially charged up to 4000 Volts and containing 800 Joules of energy. I had assumed at the outset of this paper that a coil gun would need as much energy at as high a voltage as possible. That appears not to be true. Such high voltages are giving rise to currents (800 – 1000 Amperes) far beyond the physical capacity of the wire (100 Amperes) in the coil. The high currents allow the resistor to burn off tremendous amounts of heat, perhaps unnecessarily.

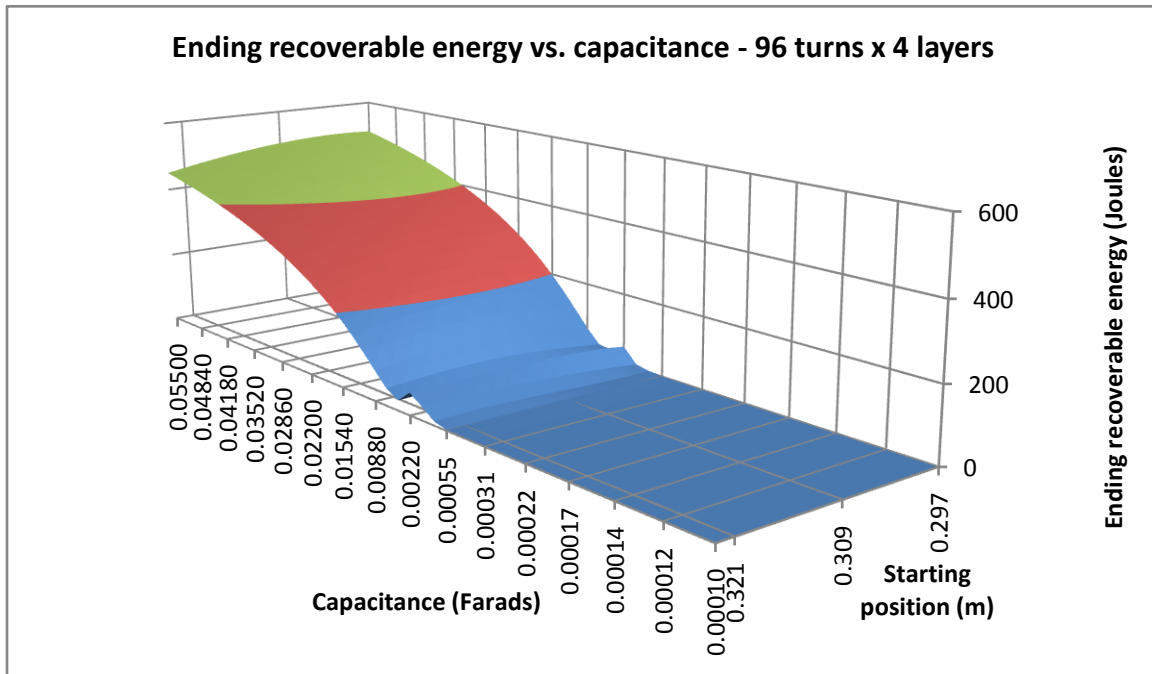
The first step is to determine if a larger capacitance, by itself, will improve the situation. For the following graph, I used a variety of capacitances and, in each case, a starting energy of 800 Joules. The base case capacitor in the previous runs was constructed from 22 physical capacitors wired in series. Each of the physical capacitors was 0.0022 F with an equivalent series resistance of 90 m Ω . For the runs shown in the graph, other composite capacitors were constructed by placing various numbers of physical capacitors in series or in parallel. The following graph shows the ending energy of the slug versus the capacitance and the slug’s starting position. The coil used in all of these runs had 96 turns per layer and four layers.



The main horizontal axis is the capacitance of the power supply. The 100 μF capacitor of the base case is at the right-hand end of this axis. The 0.055 F capacitor at the left-end of this axis is constructed from 25 of the physical wired in parallel. The depth axis shows the starting position of the slug, which varied in these runs from 29.7 cm from the center of the coil to 32.3 cm from the center of the coil.

This graph shows that the ending energy of the slug increases as the capacitance increases, up to the point where the power source is about 0.0088 F. Increasing the capacitance beyond that value tends to decrease the slug's ending energy. (The ripple seen near 0.0022 F occurs because of the timing of the oscillations in the current.)

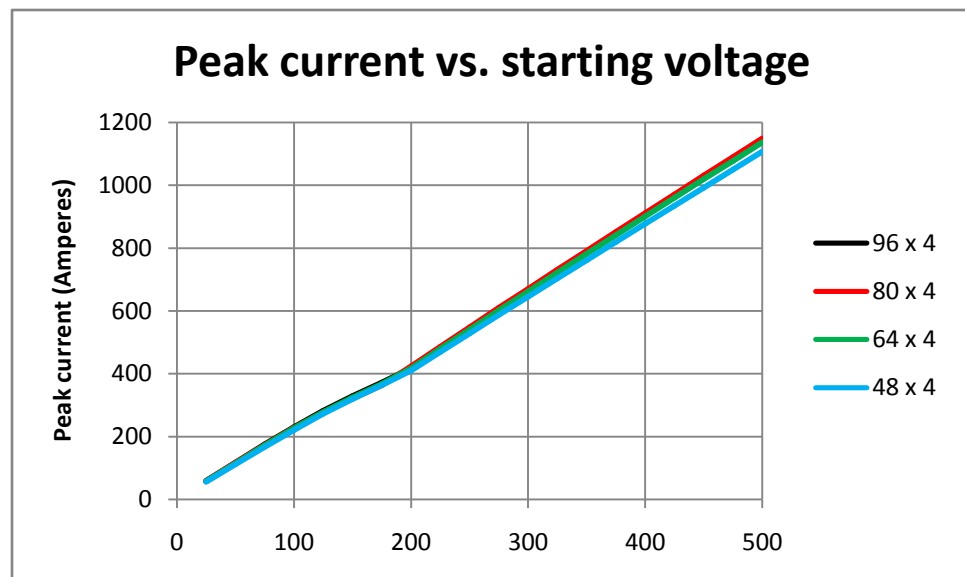
The reason why capacitance above 0.0022 F is not helpful is shown in the following graph, which shows the recoverable energy in the system at the instant when the slug passes through the center of the coil.



The “recoverable energy” is the sum of the energy stored in the capacitor’s electric field and the in coil’s magnetic field. While this is not “wasted” energy *per se*, it is very harmful to our purpose. Energy which remains in these two components is going to come out later, as current flow, after the slug has passed through the midpoint. This current will cause an attractive force which will slow down the slug. We certainly do not want that.

A second problem we encountered in Part III of this paper is that the peak current is too high. To reduce the peak current, we need to reduce the initial voltage on the capacitor. The graph at the right shows this.

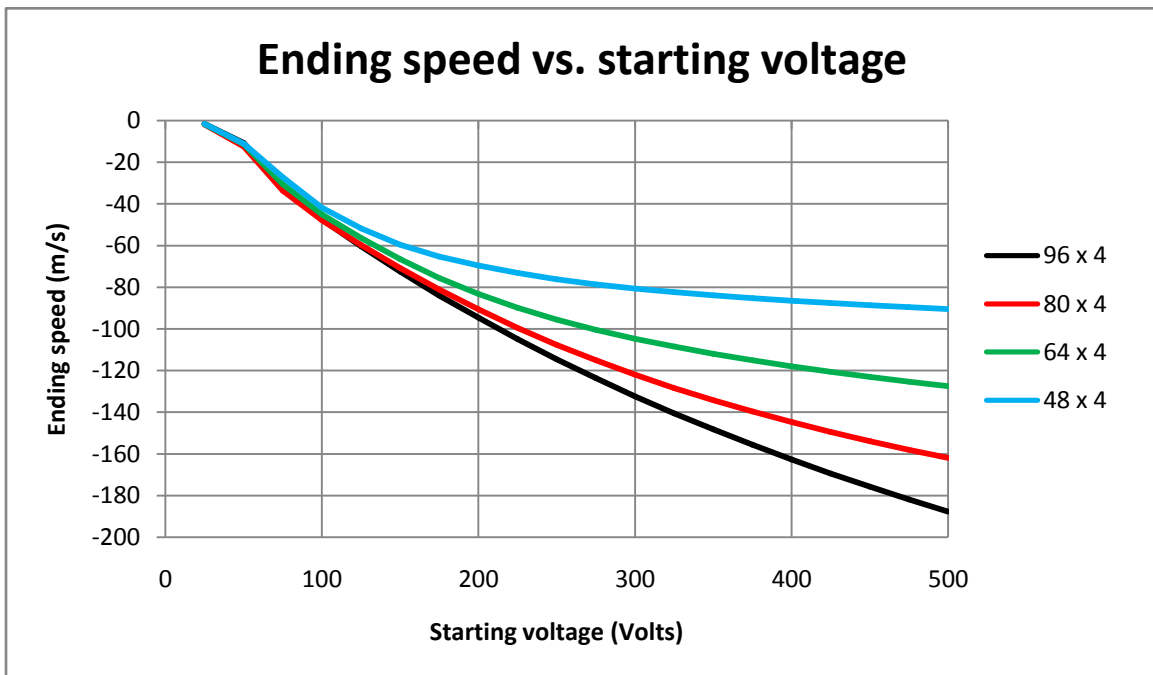
The same capacitor – 0.0088 F – was used in all of these runs. There is one curve



for each of the four-layer coils. It seems that the peak current depends only on the initial voltage, and not on the number of turns. This is not unexpected, since we chose the length of the four coils so that their one-layer versions all have the same resistance. Trying to keep the peak current below 100 Amperes is going to require very low initial voltages (at least compared with what I had expected).

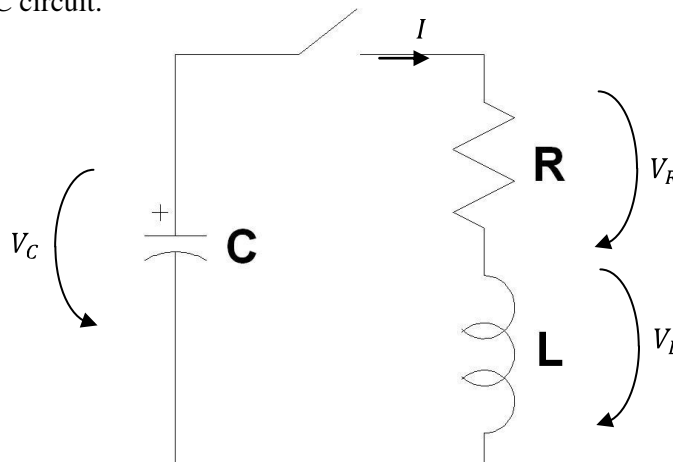
It looks as if the power supply is going to be quite a bit less than 100 Volts, perhaps only 30 – 50 Volts. This is significantly less than the kilovolt range I had anticipated. Arguably, it is getting into the range where direct battery power is feasible.

Unfortunately, low power and low current also mean low ending speed for the slug. The following graph shows the ending speed of the slug as a function of the starting voltage for the same range of voltages and for the same four coils as shown above.



Current pulse design

Before running more runs, let us see what we can say about the “perfect” current waveform. Consider the following simple R-L-C circuit.



An examination of the results for some of the runs shows that the “voltage drop over the “slug” is very small compared with the voltage drop over the Ohmic resistance at the start of a run. It remains relatively small at least up until the time when the current peaks. The “voltage drop over the slug” does not become relatively large, compared with the Ohmic voltage drop, until later in the run. Therefore, for times up until the current peak, we can ignore the “voltage drop over the slug” and use the schematic shown here with the resistor R being fixed and equal to the sum of the Ohmic resistances in the circuit.

Ignoring the slug’s nonlinear voltage drop allows us to represent the electrical circuit as a classical fixed-component R-L-C circuit. The differential equation for the current is:

$$LC \frac{d^2 I}{dt^2} + RC \frac{dI}{dt} + I = 0 \quad (1)$$

The characteristic equation, which results from assuming a solution of form $I(t) = Ae^{\alpha t}$, is:

$$LC\alpha^2 + RC\alpha + 1 = 0 \quad (2)$$

whose solution is:

$$\alpha = \frac{-RC \pm \sqrt{R^2 C^2 - 4LC}}{2LC} \quad (3)$$

When $R^2 C^2 - 4LC$ is positive, the current is the sum of two exponential terms. When $R^2 C^2 - 4LC$ is negative, the current is a sinusoid whose envelope decays exponentially. The circumstance which separates the two general types of solutions occurs when $R^2 C^2 - 4LC$ is zero.

Alert readers will see that, when $R^2 C^2 - 4LC$ is zero, there is only a single value for α , which is $\alpha = -R/2L$. However, the second-order differential equation must have two distinct solutions and there is, indeed, a second solution. It just does not happen to have the form $e^{\alpha t}$. The second solution has the form $I(t) = Bte^{\beta t}$. Substituting this form into differential equation (1) proceeds as follows:

$$\begin{aligned} & LC \frac{d}{dt} (Be^{\beta t} + Bt\beta e^{\beta t}) + RC (Be^{\beta t} + Bt\beta e^{\beta t}) + Bte^{\beta t} = 0 \\ \rightarrow & LC (B\beta e^{\beta t} + B\beta e^{\beta t} + Bt\beta^2 e^{\beta t}) + RC (Be^{\beta t} + Bt\beta e^{\beta t}) + Bte^{\beta t} = 0 \\ \rightarrow & LC (\beta + \beta + t\beta^2) + RC (1 + t\beta) + t = 0 \\ \rightarrow & t(LC\beta^2 + RC\beta + 1) + (2\beta LC + RC) = 0 \end{aligned} \quad (4)$$

Since time t can take on any value, characteristic equation (4) can hold true only if the two terms are identically equal to zero. Setting the coefficient of time t to zero produces another instance of Equation (2), which will then require that $\beta = \alpha$. Setting the constant term equal to zero as well gives:

$$\begin{aligned} & 2\beta LC + RC = 0 \\ \rightarrow & \beta = \frac{-RC}{2LC} \\ \rightarrow & \beta = \frac{-R}{2L} \end{aligned} \quad (5)$$

which is, again, $\beta = \alpha$. This confirms that $I(t) = te^{\alpha t}$ and $I(t) = e^{\alpha t}$ are both solutions of the differential equation when the term under the radical is equal to zero.

Continuing with the assumption that $R^2C^2 - 4LC = 0$, we can write the current as a function of time as:

$$I(t) = Ae^{-Rt/2L} + Bte^{-Rt/2L} \quad (6)$$

where the constants A and B depend on the particular initial state of the circuit. In our runs, the circuit starts from rest. The current at time $t = 0$ is zero, which requires that $A = 0$. Since no current flows at time $t = 0$, there will be no voltage drop over resistor R and the voltage drop over the inductor must be equal to the initial voltage drop over the capacitor $V_C(0)$. Since we know the relationship between the voltage drop over the inductor and the time-derivative of the current, we can write:

$$\begin{aligned} V_L(t) &= L \frac{dI}{dt} \\ &= L \frac{d}{dt} (Bte^{-Rt/2L}) \\ &= LB \left(e^{-Rt/2L} - \frac{R}{2L} te^{-Rt/2L} \right) \\ \rightarrow V_L(0) &= LB \\ \rightarrow V_C(0) &= LB \\ \rightarrow B &= \frac{V_C(0)}{L} \end{aligned} \quad (7)$$

Therefore, the complete waveform for the current can be written as:

$$I(t) = \frac{V_C(0)}{L} te^{-Rt/2L} \quad (8)$$

In the sections above, we came to the conclusion that we needed to restrict the peak value of the current to a fixed value, such as $I_{MaxAllowed} = 100$ Amperes. We can determine when the current described by Equation (8) reaches its maximum. We will call this time $t = t_{max}$. At the instant of the maximum, the slope of Equation (8) will be zero, thus:

$$\begin{aligned} \frac{dI}{dt} &= \frac{V_C(0)}{L} \left(1 - \frac{R}{2L} t \right) e^{-Rt/2L} \\ \rightarrow 0 &= \frac{V_C(0)}{L} \left(1 - \frac{R}{2L} t_{max} \right) e^{-Rt_{max}/2L} \\ \rightarrow t_{max} &= \frac{2L}{R} \end{aligned} \quad (9)$$

And, at this time, the magnitude of the current will be:

$$\begin{aligned} I(t_{max}) &= \frac{V_C(0)}{L} t_{max} e^{-Rt_{max}/2L} \\ &= \frac{V_C(0)}{L} \frac{2L}{R} e^{-R2L/2LR} \\ &= \frac{2V_C(0)}{eR} \end{aligned} \quad (10)$$

where the e in the denominator is the natural number. If we want the current at its peak to be equal to the maximum allowable current, then we must have:

$$V_C(0) = \frac{1}{2}eRI_{MaxAllowed} \quad (11)$$

Equation (11) is a restriction on the maximum voltage to which the capacitor can be charged before a run. If the capacitor is charged to a higher voltage, the current will overshoot the allowable peak value. Let us take a look at an example for the current waveform, using component values for the 96-turn four-layer coil in the runs above.

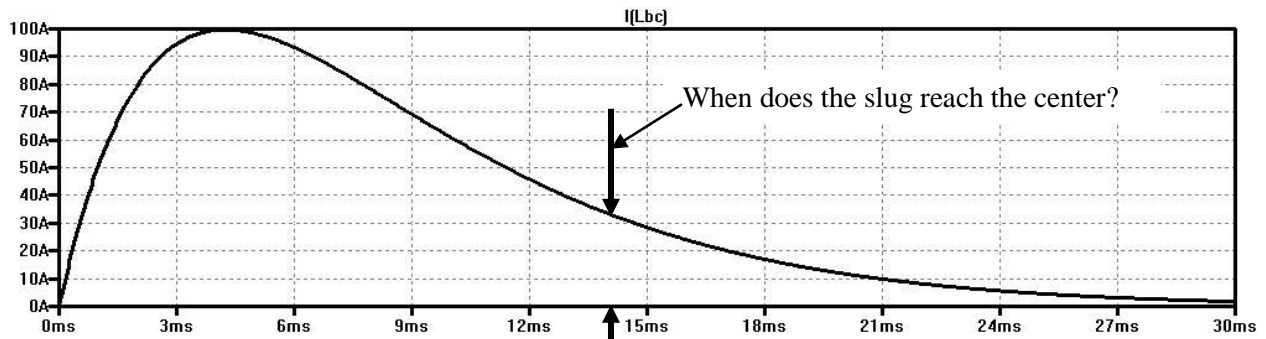
$$R = R_{esr} + R_{sw} + R_{coil} = 0.0225 + 0.15 + 0.0554 = 0.2279 \Omega \text{ and}$$

$$L = 0.0004853 \text{ H.}$$

The formulation in this section requires that $R^2C^2 - 4LC = 0$. This happens when the value of the capacitance is $C = 0.0374 \text{ F}$ and not the $C = 0.0085 \text{ F}$ used in the runs. Using the larger capacitance, and limiting the current to $I_{MaxAllowed} = 100 \text{ Amperes}$, requires that:

$$\left. \begin{aligned} V_C(0) &= \frac{1}{2}e \times 0.2279 \times 100 = 30.97 \text{ Volts} \\ t_{max} &= \frac{2 \times 0.0004853}{0.2279} = 0.00426 \text{ seconds} \end{aligned} \right\} \quad (12)$$

The current waveform looks like this:



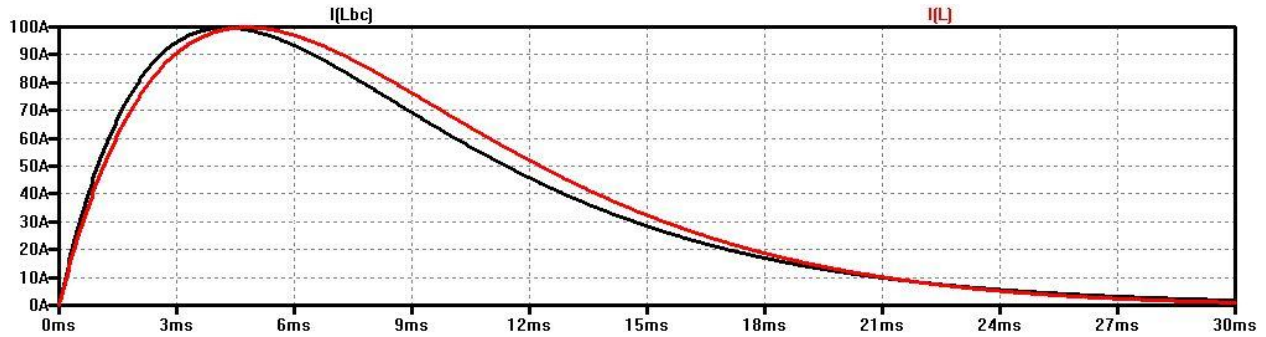
The current peaks at 100 Amperes at 4.26 ms, as planned. However, there is another matter we have not been focusing on. When does the slug reach the center of the coil? And, more importantly, what is the current when the slug gets there?

We cannot permit the current to flow after the slug reaches the center of the coil. That would slow it down. It seems like we need to specify two restrictions on the waveform: a maximum peak current and a maximum allowable current when the slug reaches the center of the coil.

Setting a target for the ending current would be satisfactory if we were confident that the current would always be positive. A more general target would apply to the combined energies in the capacitor and inductor. In previous parts of this paper, I have called this the “recoverable energy” at the instant when the slug reaches the center of the coil. This is energy which will be released, in the form of current, while the slug is in the right-hand side of the coil on its way out of the coil gun.

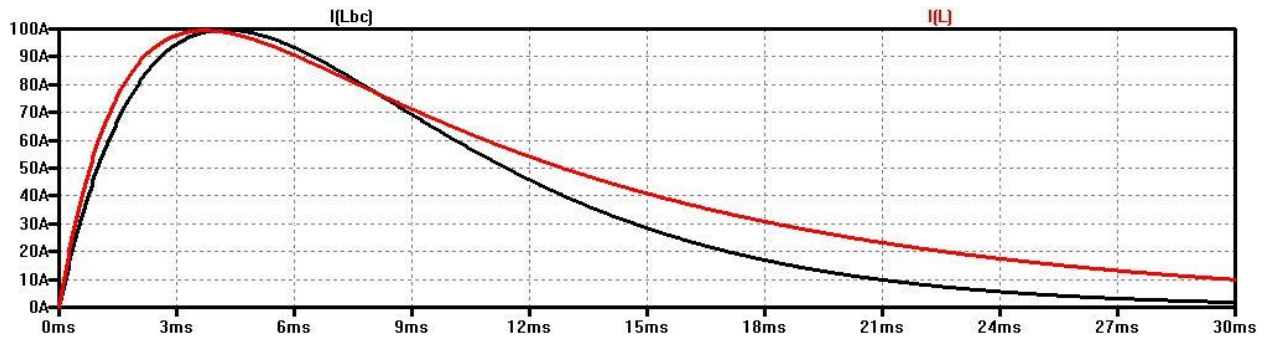
In the runs which follow, I have set this target to one percent of the energy stored in the capacitor at the start of the run.

The question to be asked is: are there ways to reduce the recoverable energy? Let us look at some possibilities. One could reduce the inductance in the circuit by adding a separator inductor in parallel with the coil, but this would divide the current between the two coils, and would result in less current for accelerating the slug. Or, one could increase the inductance by adding a separate inductor in series with the coil, but this tends to increase the recoverable energy, not reduce it. The following graph compares the base case (in black) with the current (plotted in red) when the inductance is increased by 20% from 0.0004853 H to 0.0005824 H. To keep the peak current equal to 100 Amperes, the starting voltage is increased to 32 Volts.



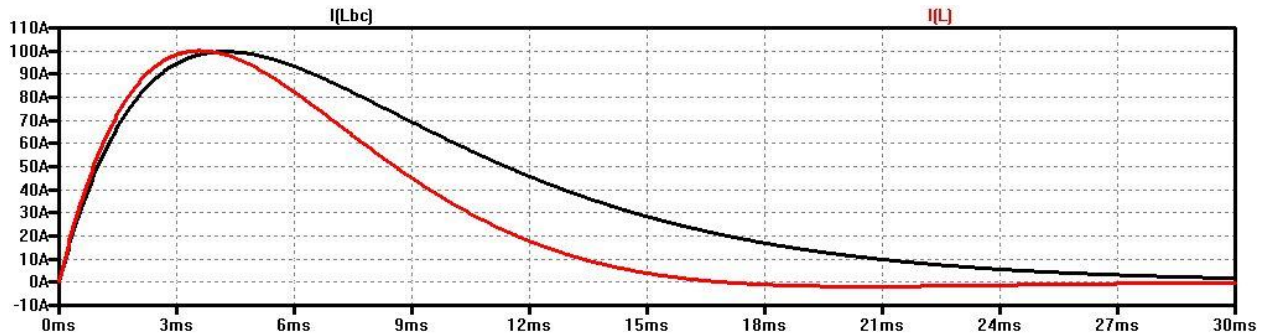
As I say, increasing the inductance does not help. It increases the current at times after the peak. So, let us try modifying the resistance.

It is impractical to decrease the resistance in the circuit. The resistance R is not a discrete component to which a separate resistor can be placed in parallel. However, it is possible to add resistance to the circuit. The following graph compares the base case (in black) with the current (plotted in red) when a 0.1 Ω resistor is added in series to the circuit and, to keep the peak current equal to 100 Amperes, the starting voltage is increased to 40 Volts.



Increasing the resistance does not work either. Its effect in lengthening the RC time-constant is greater than its effect in decreasing the RL time-constant. So, let us try reducing the capacitor.

The following graph compares the base case (in black) with the current (plotted in red) when the capacitance is reduced by 20% from 0.0374 F to 0.02992 F and, to keep the peak current equal to 100 Amperes, the starting voltage is increased to 34 Volts.



This works. Decreasing the capacitance reduces the current at all times following the peak. With enough decrease in the capacitance, the current at the instant when the slug reaches the center of the coil can be reduced below any pre-set target value.

Optimized results for the four coils with four layers

In this section, we will examine the “best case” configuration for the four coils with four layers. Finding the best case for any such coil is an iterative process involving variations in three independent parameters:

- the value of the capacitance,
- the starting voltage on the capacitor and
- the starting position for the slug.

For any given capacitor and starting position, the starting voltage is adjusted to the magnitude which gives a peak current of 100 Amperes. The ending energy of the slug is compared for the various starting positions, and the starting position which gives the highest ending speed is noted. However, if the recoverable energy is greater than one percent of the capacitor’s starting energy, then a smaller capacitor is tried.

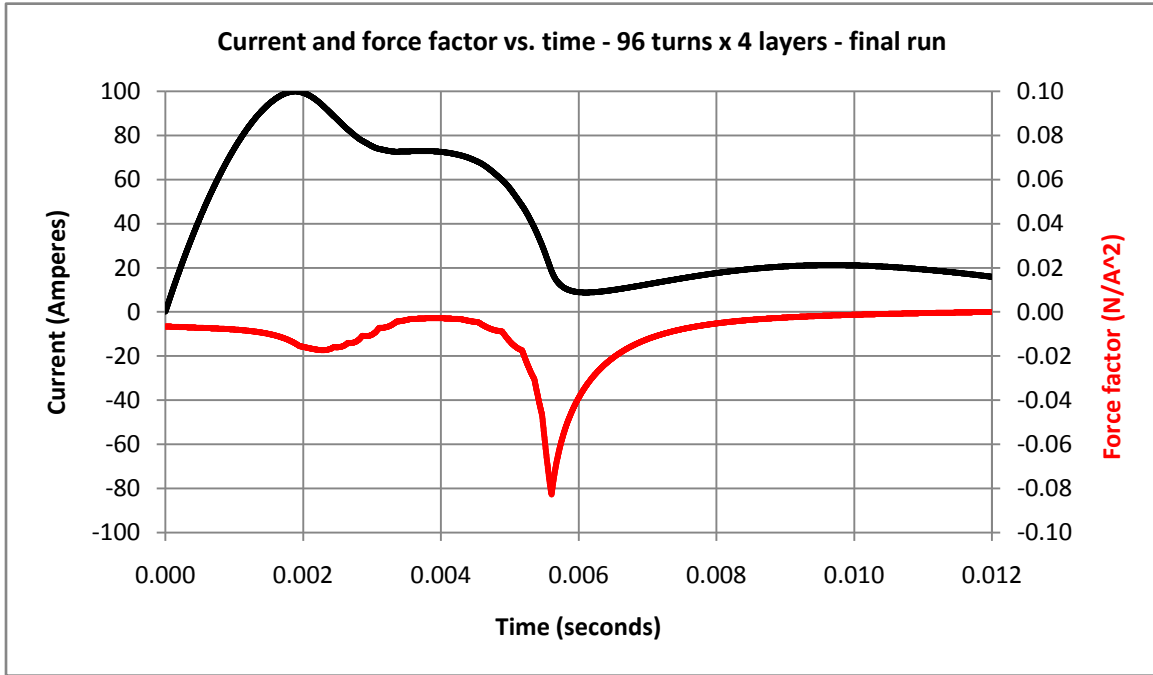
For these runs, the capacitor was a composite capacitor consisting of several of the standard component capacitors – 0.0022 F with $R_{esr} = 0.09 \Omega$ – wired in parallel. Here are the bet cases identified by numerical simulation:

Coil →	96 turns x 4 layers	80 turns x 4 layers	64 turns x 4 layers	48 turns x 4 layers
Capacitor	0.011 F	0.011 F	0.011 F	0.0154 F
Initial voltage	46.500 V	45.666 V	45.220 V	41.439 V
Initial energy	11.892 J	11.470 J	11.247 J	13.222 J
Starting position	0.2895 m	0.2523 m	0.2170 m	0.1868 m
Ending speed	31.189 m/s	30.093 m/s	28.861 m/s	29.520 m/s
Slug’s ending energy	4.858 J	4.522 J	4.160 J	4.352 J
Efficiency	40.85%	39.43%	36.99%	32.91%

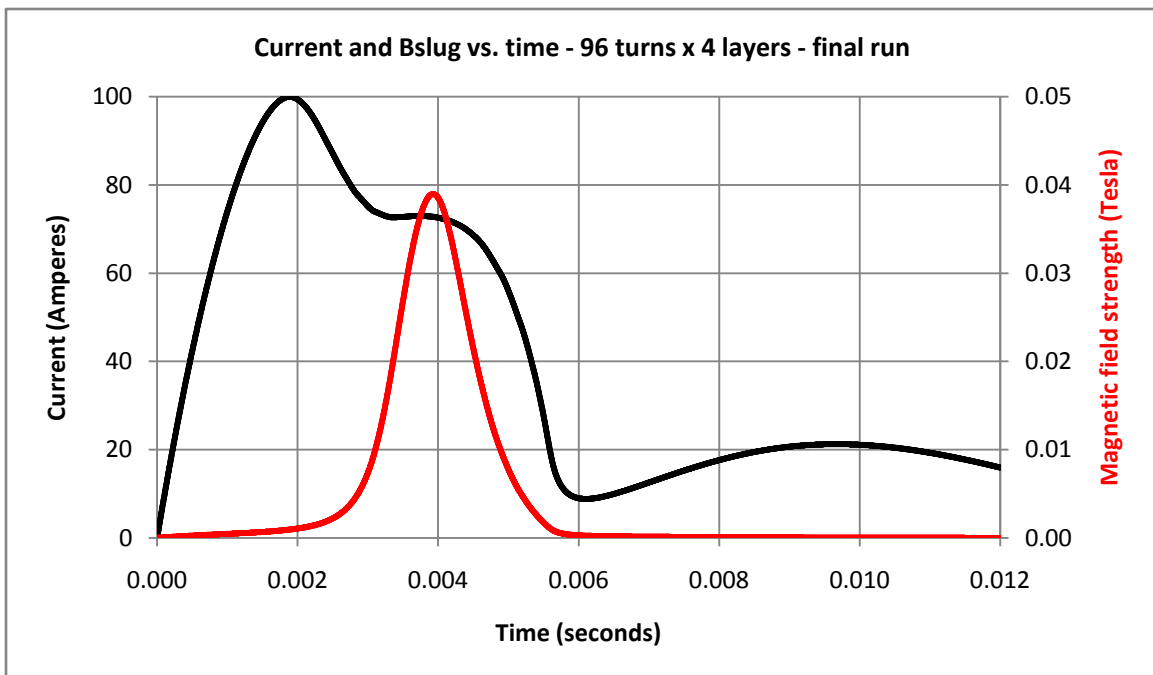
The 96-turn, 80-turn and 64-turn coils are powered by five of the physical capacitors wired in parallel. The 48-turn coil can use seven such capacitors in parallel. The initial voltages range between 41 and 47 Volts, with initial energies roughly between 11 and 13 Joules. This is far, far less than the 4000 Volts and 800 Joules I had imagined at the outset of this paper. The ending speeds of the slug range between 29 meters per second and 31 meters per second. And, between 30% and 40% of the initial energy in the capacitor is converted into kinetic energy of the slug.

Overall, the longer coil delivers slightly better performance than the others. It seems to deliver more energy, and deliver it more efficiency, than the other coils. However, the outcomes from the runs are very sensitive to a number of factors, and it would be premature to conclude that longer coils are better.

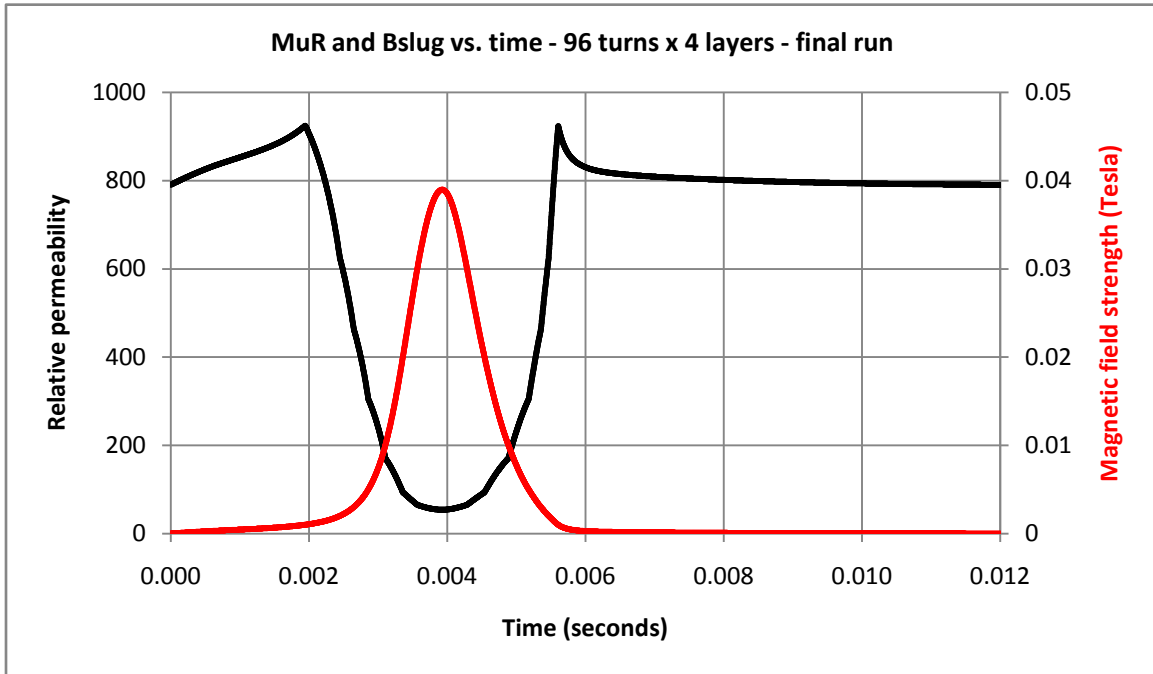
The following graphs show the details during the run for the 96-turn four layer coil. The parameters for this run are those set out in the left-most numerical column in the table above. I have labeled this as the “final run” because I do not believe that much better results are available from the coils we have at hand. The first graph shows the current and the force factor during the run. The slug reaches the center of the coil at time $t = 12.007$ ms, just to the right of the end of the horizontal axis. The current flowing is still significant – 15.91 Amperes – but the combined energy in the capacitor and the inductor is nevertheless less than one percent of the starting energy.



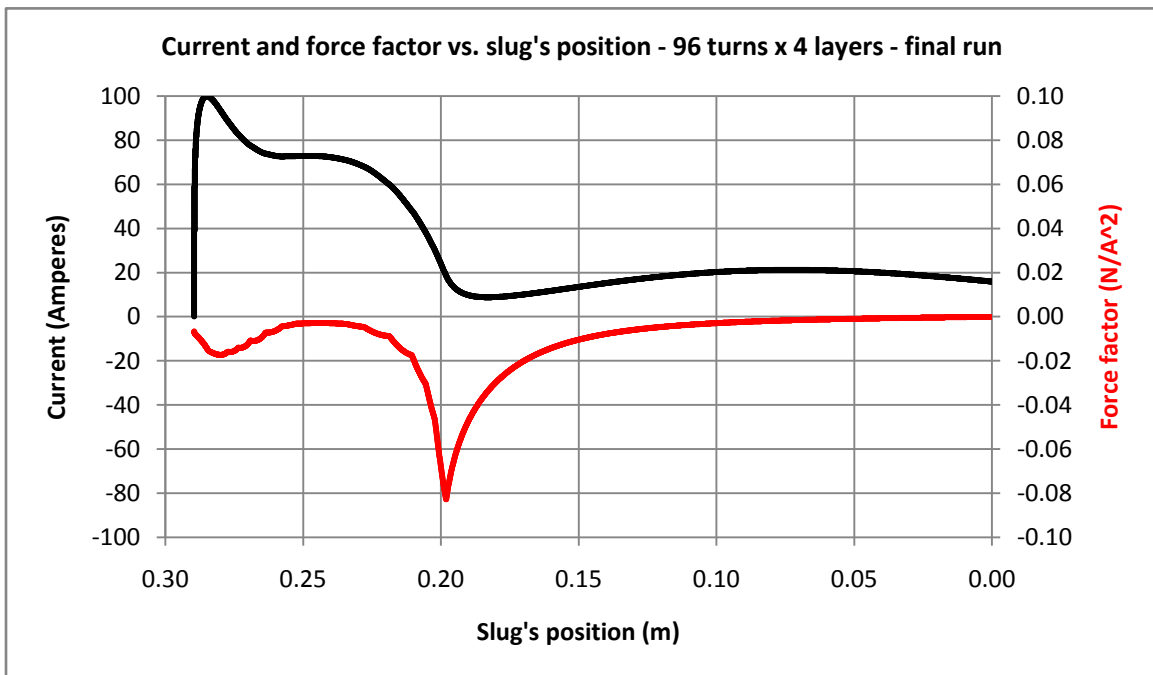
The next graph shows the current and the applied magnetic field strength (at the center of the slug) with respect to time.



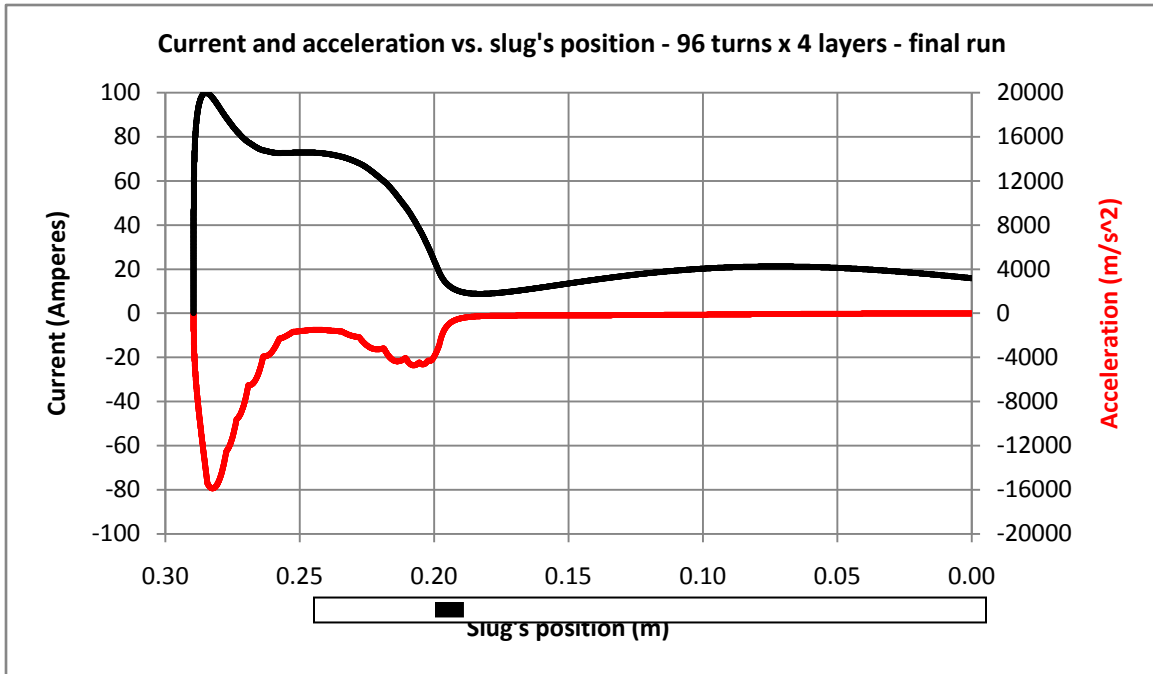
The relative permeability of the slug, which is so very sensitive to the strength of the applied magnetic field, is plotted with respect to time in the next graph.



The following graph shows the current and the force factor once again, this time plotted with respect to the slug's position.



The final graph shows the current and the slug's acceleration, both with respect to the position of the slug during the run. For reference, I have shown a scale outline of the coil and slug in the figure.



The slug starts from a position located about 29 cm from the center of the coil and is accelerated over a total distance of about 9 cm. Its acceleration is greatest in two regions located two or three centimeters on either side of the face of the coil. In fact, the slug's acceleration is less when it is at the face than when it is two or three centimeters from the face, notwithstanding that the magnetic field is greatest at the face. The slug is saturated at the face and cannot respond to the applied magnetic field as well as one might hope. In a sense, it seems like we are wasting something. It could be that a lower current would actually result in a greater force / acceleration on the slug. That is a possibility that needs more study.

In the next part of this paper, we will examine two new things: the effect of a bigger slug and the effect of giving the slug an initial speed.

Jim Hawley
September 2012

An e-mail setting out errors and omissions would be appreciated.

Appendix “A” – Listing of Visual Basic Sub-programs

The results presented in this part of the paper were produced using the Visual Basic program *Integration6*, a listing of which was given in the Appendices of Part IV. The control form, *Form1*, in the earlier listing included three Sub-programs which produced the results given in Part III of this paper. To produce the results above, three more Sub-programs were placed in the control *Form*. The following listing gives these three additional Sub-programs.

```
'////////////////////////////////////
'////////////////////////////////////
'// Use this block to integrate a selection of runs
'// using different capacitor sizes.
'// Sub-program #4
'// This sub-program produced results for Part IV of the paper.
labelDisplay.Visible = True
Dim InputFileName As String
Dim OutputFileName As String
Dim RowNum As Int32 ' Row index in the output spreadsheet
Dim Z0mid As Double ' Midpoint of starting values of Z0
Dim EC0 As Double
' Open the Excel file for the results.
' The file must exist before execution begins.
OutputFileName = "C:\Integration6Results-DifferentCapacitors.xlsx"
Try
    objExcel = CType(CreateObject("Excel.Application"), _
        Microsoft.Office.Interop.Excel.Application)
    objExcel.Visible = False
    objExcelWB = CType(objExcel.Workbooks.Open(OutputFileName), _
        Microsoft.Office.Interop.Excel.Workbook)
    objExcelWS = CType(objExcelWB.Sheets("Sheet1"), _
        Microsoft.Office.Interop.Excel.Worksheet)
Catch ex As Exception
    Cursor.Current = Cursors.Default
    MsgBox("Could not open the Excel file to save the output.", vbOKOnly)
Exit Sub
End Try
With objExcelWS
    .Cells(1, 1) = "Integration #6 results for different capacitor values."
End With
RowNum = 2
' Initialize the relative permeability table.
InitializeMuTable()
' Main loop to step through four coils.
For I As Int32 = 1 To 1 Step 1
    Select Case I
        Case 1
            Nturns = 96
            Z0mid = 0.309
        Case 2
            Nturns = 80
            Z0mid = 0.272
        Case 3
            Nturns = 64
            Z0mid = 0.2385
        Case 4
            Nturns = 48
            Z0mid = 0.21
    End Select
End For
```

```

End Select
Nlayers = 4
' Read the file with the spatial distribution of the force.
' This call will set the following parameters:
'   Rcore, Dwire, Nturns, Nlayers
'   FnumZ, FleftZ, FrightZ
InputFileName = "C:\FieldTextFile-" & Trim(Str(Nlayers)) & _
    "x" & Trim(Str(Nturns)) & "TurnsWith1201x399.txt"
ReadAndCalculateSpatialDistribution(InputFileName)
' Write the configuration header.
RowNum = RowNum + 2
With objExcelWS
    .Cells(RowNum, 1) = "File name"
    .Cells(RowNum + 1, 1) = "DeLT (s)"
    .Cells(RowNum + 2, 1) = "Rcore (m)"
    .Cells(RowNum + 3, 1) = "Dwire (m)"
    .Cells(RowNum + 4, 1) = "Nturns"
    .Cells(RowNum + 5, 1) = "Nlayers"
    .Cells(RowNum + 6, 1) = "Hcoil (m)"
    .Cells(RowNum + 7, 1) = "FnumZ"
    .Cells(RowNum + 8, 1) = "FleftZ (m)"
    .Cells(RowNum + 9, 1) = "FrightZ (m)"
    .Cells(RowNum + 10, 1) = "gridDelZ (m)"
    .Cells(RowNum + 11, 1) = "Rcoil (Ohms)"
    .Cells(RowNum + 12, 1) = "Lcoil (H)"
    .Cells(RowNum + 13, 1) = "RperM (O/m)"
    .Cells(RowNum + 14, 1) = "Rslug (m)"
    .Cells(RowNum + 15, 1) = "Hslug (m)"
    .Cells(RowNum + 16, 1) = "RHslug (k/m3)"
    .Cells(RowNum + 17, 1) = "Mass (kg)"
    .Cells(RowNum + 18, 1) = "EC0 (J)"
    .Cells(RowNum + 19, 1) = "Rsw (Ohms)"
End With
' Set the parameters which are fixed during all runs.
delT = 0.000001      ' The time step for the integration is 100 ns
Rslug = 0.0045      ' The slug has a 9 mm diameter
Hslug = 0.02        ' The slug is 20 mm long
RHslug = 7850      ' Steel weighs 7850 kg/m^3
M = Math.PI * Rslug * Rslug * Hslug * RHslug ' Mass of the slug
RPerM = 0.00081543 ' Linear resistance of the wire
EC0 = 800          ' Initial energy in the capacitor
Rsw = 0.15        ' Static resistance of the switch
' Calculate the length of the coil.
Hcoil = Dwire * Nturns
' Calculate the resistance and inductance of the coil.
BuildTheCoil()
' Calculate the total circuit parameters.
L = Lcoil
' Write the fixed parameters in the header.
With objExcelWS
    .Cells(RowNum, 2) = InputFileName
    .Cells(RowNum + 1, 2) = Trim(Str(delT))
    .Cells(RowNum + 2, 2) = Trim(Str(Rcore))
    .Cells(RowNum + 3, 2) = Trim(Str(Dwire))
    .Cells(RowNum + 4, 2) = Trim(Str(Nturns))
    .Cells(RowNum + 5, 2) = Trim(Str(Nlayers))
    .Cells(RowNum + 6, 2) = Trim(Str(Hcoil))
    .Cells(RowNum + 7, 2) = Trim(Str(FnumZ))

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.Cells(RowNum + 8, 2) = Trim(Str(FleftZ))
.Cells(RowNum + 9, 2) = Trim(Str(FrightZ))
.Cells(RowNum + 10, 2) = Trim(Str((FleftZ - FrightZ) / (FnumZ - 1)))
.Cells(RowNum + 11, 2) = Trim(Str(Rcoil))
.Cells(RowNum + 12, 2) = Trim(Str(Lcoil))
.Cells(RowNum + 13, 2) = Trim(Str(RPerM))
.Cells(RowNum + 14, 2) = Trim(Str(Rslug))
.Cells(RowNum + 15, 2) = Trim(Str(Hslug))
.Cells(RowNum + 16, 2) = Trim(Str(RH0slug))
.Cells(RowNum + 17, 2) = Trim(Str(M))
.Cells(RowNum + 18, 2) = Trim(Str(EC0))
.Cells(RowNum + 19, 2) = Trim(Str(Rsw))
.Cells(RowNum + 21, 1) = "Ending Slug's Energy (J)"
.Cells(RowNum + 22, 1) = "Z0 (m)"           ' Starting position of slug
.Cells(RowNum + 23, 1) = "C (F)"           ' Capacitance
.Cells(RowNum + 23, 2) = "Resr (Ohms)"      ' Capacitor's ESR
.Cells(RowNum + 21 + 55, 1) = "Ending Recoverable Energy (J)"
.Cells(RowNum + 22 + 55, 1) = "Z0 (m)"     ' Starting position of slug
.Cells(RowNum + 23 + 55, 1) = "C (F)"     ' Capacitance
.Cells(RowNum + 23 + 55, 2) = "Resr (Ohms)" ' Capacitor's ESR
End With
' Main loop to step through different capacitance values.
' We will use 1 to 22 of our 2200uF standard capacitors in series.
' Therefore, Ctotal = 0.0022 F / number of capacitors.
' The series resistance of each capacitor is 90mOhms.
' Therefore, Resr = 0.090 Ohms * number of capacitors.
For J As Int32 = 1 To 46 Step 1
  If (J <= 22) Then
    ' Capacitors in series
    C = 0.0022 / (23 - J)
    VC0 = Math.Sqrt(2 * EC0 / C)
    QC0 = C * VC0
    Resr = 0.09 * (23 - J)
    R = Resr + Rsw + Rcoil
  Else
    ' Capacitors in parallel
    C = 0.0022 * (J - 21)
    VC0 = Math.Sqrt(2 * EC0 / C)
    QC0 = C * VC0
    Resr = 0.09 / (J - 21)
    R = Resr + Rsw + Rcoil
  End If
  ' Main loop to step through different starting positions.
  For K As Int32 = -7 To +7 Step 1
    Z0 = Z0mid + (K * 0.002)
    ' Execute the run.
    IntegrateOneRun(False, objExcelWS, 0)
    ' Write the results to the Excel file.
    With objExcelWS
      .Cells(RowNum + 22, 9 + K) = Trim(Str(Z0))
      .Cells(RowNum + 23 + J, 1) = Trim(Str(C))
      .Cells(RowNum + 23 + J, 2) = Trim(Str(Resr))
      .Cells(RowNum + 23 + J, 9 + K) = Trim(Str(ESend))
      .Cells(RowNum + 22 + 55, 9 + K) = Trim(Str(Z0))
      .Cells(RowNum + 23 + 55 + J, 1) = Trim(Str(C))
      .Cells(RowNum + 23 + 55 + J, 2) = Trim(Str(Resr))
      .Cells(RowNum + 23 + 55 + J, 9 + K) = Trim(Str(ECend + ELeND))
    End With
  End With

```

```

        Next K
    Next J
    RowNum = RowNum + 133
Next I
' Re-integrate from the best starting position and,
' this time, show the details of the run.
'Z0 = Z0mid
'IntegrateOneRun(True, objExcelWS, 35)
' Save and close the output file.
objExcelWB.Save()
objExcelWB.Close()
objExcel.Quit()
MsgBox("All done.")
'////////////////////////////////////
'////////////////////////////////////

'////////////////////////////////////
'////////////////////////////////////
'// Use this block to integrate a selection of runs
'// using different starting voltages for a single
'// capacitor.
'// Sub-program #5
'// This sub-program produced results for Part IV of the paper.
labelDisplay.Visible = True
Dim InputFilename As String
Dim OutputFileName As String
Dim RowNum As Int32 ' Row index in the output spreadsheet
Dim Z0mid As Double ' Midpoint of starting values of Z0
Dim EC0 As Double ' Starting energy in capacitor
' Open the Excel file for the results.
' The file must exist before execution begins.
OutputFileName = "C:\Integration6Results-DifferentVoltages.xlsx"
Try
    objExcel = CType(CreateObject("Excel.Application"), _
        Microsoft.Office.Interop.Excel.Application)
    objExcel.Visible = False
    objExcelWB = CType(objExcel.Workbooks.Open(OutputFileName), _
        Microsoft.Office.Interop.Excel.Workbook)
    objExcelWS = CType(objExcelWB.Sheets("Sheet1"), _
        Microsoft.Office.Interop.Excel.Worksheet)
Catch ex As Exception
    Cursor.Current = Cursors.Default
    MsgBox("Could not open the Excel file to save the output.", vbOKOnly)
Exit Sub
End Try
With objExcelWS
    .Cells(1, 1) = "Integration #6 results for different starting voltages."
End With
' Initialize the relative permeability table.
InitializeMuTable()
' Set the parameters which are fixed during all runs.
delT = 0.0000001 ' The time step for the integration is 100 ns
Rslug = 0.0045 ' The slug has a 9 mm diameter
Hslug = 0.02 ' The slug is 20 mm long
RHOSlug = 7850 ' Steel weighs 7850 kg/m^3
M = Math.PI * Rslug * Rslug * Hslug * RHOSlug ' Mass of the slug
RPerM = 0.00081543 ' Linear resistance of the wire
Rsw = 0.15 ' Static resistance of the switch

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' Write the fixed parameters to the file.
With objExcelWS
    .Cells(3, 1) = "DeLT = " & Trim(Str(deLT)) & " (s)"
    .Cells(4, 1) = "Rslug = " & Trim(Str(Rslug)) & " (m)"
    .Cells(5, 1) = "Hslug = " & Trim(Str(Hslug)) & " (m)"
    .Cells(6, 1) = "RH0slug = " & Trim(Str(RH0slug)) & " (k/m3)"
    .Cells(7, 1) = "Mass = " & Trim(Str(M)) & " (kg)"
    .Cells(8, 1) = "RperM = " & Trim(Str(RPerM)) & " (Ohms/m)"
    .Cells(9, 1) = "Rsw = " & Trim(Str(Rsw)) & " (Ohms)"
End With
RowNum = 11
' Main loop to step through four coils.
For I As Int32 = 1 To 4 Step 1
    Select Case I
        Case 1
            Nturns = 96
            Z0mid = 0.309
        Case 2
            Nturns = 80
            Z0mid = 0.271
        Case 3
            Nturns = 64
            Z0mid = 0.239
        Case 4
            Nturns = 48
            Z0mid = 0.211
    End Select
    Nlayers = 4
    ' Read the file with the spatial distribution of the force.
    ' This call will set the following parameters:
    ' Rcore, Dwire, Nturns, Nlayers
    ' FnumZ, FleftZ, FrightZ
    InputFilename = "C:\FieldTextFile-" & Trim(Str(Nlayers)) & _
        "x" & Trim(Str(Nturns)) & "TurnsWith1201x399.txt"
    ReadAndCalculateSpatialDistribution(InputFilename)
    ' Calculate the length of the coil.
    Hcoil = Dwire * Nturns
    ' Calculate the resistance and inductance of the coil.
    BuildTheCoil()
    ' Calculate the total circuit parameters.
    L = Lcoil
    ' Write the coil parameters to the file.
    With objExcelWS
        .Cells(RowNum, 1) = "Rcore = " & Trim(Str(Rcore)) & " (m)"
        .Cells(RowNum + 1, 1) = "Dwire = " & Trim(Str(Dwire)) & " (m)"
        .Cells(RowNum + 2, 1) = "Nturns = " & Trim(Str(Nturns))
        .Cells(RowNum + 3, 1) = "Nlayers = " & Trim(Str(Nlayers))
        .Cells(RowNum + 4, 1) = "Hcoil = " & Trim(Str(Hcoil)) & " (m)"
        .Cells(RowNum + 5, 1) = "FnumZ = " & Trim(Str(FnumZ))
        .Cells(RowNum + 6, 1) = "FleftZ = " & Trim(Str(FleftZ)) & " (m)"
        .Cells(RowNum + 7, 1) = "FrightZ = " & Trim(Str(FrightZ)) & " (m)"
        .Cells(RowNum + 8, 1) = "Rcoil = " & Trim(Str(Rcoil)) & " (Ohms)"
        .Cells(RowNum + 9, 1) = "Lcoil = " & Trim(Str(Lcoil)) & " (H)"
        .Cells(RowNum + 10, 1) = "Zstart = " & Trim(Str(Z0mid)) & " (m)"
        .Cells(RowNum + 11, 1) = "C (F)"
        .Cells(RowNum + 11, 2) = "Resr (O)"
        .Cells(RowNum + 11, 3) = "EC0 (J)"
        .Cells(RowNum + 11, 4) = "VC0 (V)"
    End With
Next I

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        .Cells(RowNum + 11, 5) = "Imax (A)"
        .Cells(RowNum + 11, 6) = "Send (m/s)"
        .Cells(RowNum + 11, 7) = "Erec (J)"
    End With
    ' Specify the capacitor in terms of the standard capacitor.
    ' Use 1 to 22 of our 2200uF standard capacitors in series or
    ' in parallel. The standard capacitor has Resr = 0.090 Ohms.
    Dim Jcap As Int32 = 25
    If (Jcap <= 22) Then
        ' Capacitors in series
        C = 0.0022 / (23 - Jcap)
        Resr = 0.09 * (23 - Jcap)
        R = Resr + Rsw + Rcoil
    Else
        ' Capacitors in parallel
        C = 0.0022 * (Jcap - 21)
        Resr = 0.09 / (Jcap - 21)
        R = Resr + Rsw + Rcoil
    End If
    ' Main loop to step through different starting voltages.
    For Jvolt As Int32 = 1 To 20 Step 1
        VC0 = 525 - (25 * Jvolt)
        EC0 = 2 * C * VC0 * VC0
        QC0 = C * VC0
        ' Set starting position.
        Z0 = Z0mid
        ' Execute the run.
        IntegrateOneRun(False, objExcelWS, 0)
        ' Write the results to the file.
        With objExcelWS
            .Cells(RowNum + 11 + Jvolt, 1) = C
            .Cells(RowNum + 11 + Jvolt, 2) = Resr
            .Cells(RowNum + 11 + Jvolt, 3) = EC0
            .Cells(RowNum + 11 + Jvolt, 4) = VC0
            .Cells(RowNum + 11 + Jvolt, 5) = MaxCurrent
            .Cells(RowNum + 11 + Jvolt, 6) = Send
            .Cells(RowNum + 11 + Jvolt, 7) = ECend + ELend
        End With
    Next Jvolt
    RowNum = RowNum + 11 + 22 + 2
Next I
' Re-integrate from the best starting position and,
' this time, show the details of the run.
'C = 0.0022 * 4
'Resr = 0.09 / 4
'R = Resr + Rsw + Rcoil
'EC0 = 13.36146
'VC0 = Math.Sqrt(2 * EC0 / C)
'QC0 = C * VC0
'Z0 = Z0mid
'IntegrateOneRun(True, objExcelWS, 35)
' Save and close the output file.
objExcelWB.Save()
objExcelWB.Close()
objExcel.Quit()
MsgBox("All done.")
'////////////////////////////////////
'////////////////////////////////////

```

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'////////////////////////////////////
'////////////////////////////////////
'// Use this block to integrate a selection of runs
'// to search for the best starting voltages for
'// different capacitors.
'// Sub-program #6
'// This sub-program produced results for Part IV of the paper.
labelDisplay.Visible = True
Dim InputFileName As String
Dim OutputFileName As String
Dim RowNum As Int32 ' Row index in the output spreadsheet
Dim Z0mid As Double ' Midpoint of starting values of Z0
Dim EC0 As Double ' Starting energy in capacitor
' Open the Excel file for the results.
' The file must exist before execution begins.
OutputFileName = "C:\Integration6Results-DifferentVIPairs.xlsx"
Try
    objExcel = CType(CreateObject("Excel.Application"), _
        Microsoft.Office.Interop.Excel.Application)
    objExcel.Visible = False
    objExcelWB = CType(objExcel.Workbooks.Open(OutputFileName), _
        Microsoft.Office.Interop.Excel.Workbook)
    objExcelWS = CType(objExcelWB.Sheets("Sheet1"), _
        Microsoft.Office.Interop.Excel.Worksheet)
Catch ex As Exception
    Cursor.Current = Cursors.Default
    MsgBox("Could not open the Excel file to save the output.", vbOKOnly)
    Exit Sub
End Try
With objExcelWS
    .Cells(1, 1) = "Integration #6 results for different starting voltages."
End With
' Initialize the relative permeability table.
InitializeMuTable()
' Set the parameters which are fixed during all runs.
delT = 0.000001 ' The time step for the integration is 100 ns
Rslug = 0.0045 ' The slug has a 9 mm diameter
Hslug = 0.02 ' The slug is 20 mm long
RHOSlug = 7850 ' Steel weighs 7850 kg/m^3
M = Math.PI * Rslug * Rslug * Hslug * RHOSlug ' Mass of the slug
RPerM = 0.00081543 ' Linear resistance of the wire
Rsw = 0.15 ' Static resistance of the switch
' Write the fixed parameters to the file.
With objExcelWS
    .Cells(3, 1) = "DelT = " & Trim(Str(delT)) & " (s)"
    .Cells(4, 1) = "Rslug = " & Trim(Str(Rslug)) & " (m)"
    .Cells(5, 1) = "Hslug = " & Trim(Str(Hslug)) & " (m)"
    .Cells(6, 1) = "RHOSlug = " & Trim(Str(RHOSlug)) & " (k/m3)"
    .Cells(7, 1) = "Mass = " & Trim(Str(M)) & " (kg)"
    .Cells(8, 1) = "RperM = " & Trim(Str(RPerM)) & " (Ohms/m)"
    .Cells(9, 1) = "Rsw = " & Trim(Str(Rsw)) & " (Ohms)"
End With
RowNum = 9
' Main loop to step through four coils.
Dim VC0start As Double
Dim NumCap As Int32
For I As Int32 = 1 To 4 Step 1

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Select Case I
  Case 1
    Nturns = 96
    Z0mid = 0.2895      ' Best case = 0.2895
    VC0start = 46.5    ' Best case = 46.50
    NumCap = 5         ' Best case = 0.011 (5)
  Case 2
    Nturns = 80
    Z0mid = 0.25225    ' Best case = 0.25225
    VC0start = 45.66593 ' Best case = 45.66593
    NumCap = 5         ' Best case = 0.011 (5)
  Case 3
    Nturns = 64
    Z0mid = 0.217      ' Best case = 0.217
    VC0start = 45.22   ' Best case = 45.22
    NumCap = 5         ' Best case = 0.011 (5)
  Case 4
    Nturns = 48
    Z0mid = 0.18675    ' Best case = 0.18675
    VC0start = 41.43895 ' Best case = 41.43895
    NumCap = 7         ' Best case = 0.0154 (7)
End Select
Nlayers = 4
' Read the file with the spatial distribution of the force.
' This call will set the following parameters:
'   Rcore, Dwire, Nturns, Nlayers
'   FnumZ, FleftZ, FrightZ
InputFilename = "C:\FieldTextFile-" & Trim(Str(Nlayers)) & _
  "x" & Trim(Str(Nturns)) & "TurnsWith1201x399.txt"
ReadAndCalculateSpatialDistribution(InputFilename)
' Calculate the length of the coil.
Hcoil = Dwire * Nturns
' Calculate the resistance and inductance of the coil.
BuildTheCoil()
' Calculate the total circuit parameters.
L = Lcoil
' Write the coil parameters to the file.
RowNum = RowNum + 2
With objExcelWS
  .Cells(RowNum, 1) = "Rcore = " & Trim(Str(Rcore)) & " (m)"
  .Cells(RowNum + 1, 1) = "Dwire = " & Trim(Str(Dwire)) & " (m)"
  .Cells(RowNum + 2, 1) = "Nturns = " & Trim(Str(Nturns))
  .Cells(RowNum + 3, 1) = "Nlayers = " & Trim(Str(Nlayers))
  .Cells(RowNum + 4, 1) = "Hcoil = " & Trim(Str(Hcoil)) & " (m)"
  .Cells(RowNum + 5, 1) = "FnumZ = " & Trim(Str(FnumZ))
  .Cells(RowNum + 6, 1) = "FleftZ = " & Trim(Str(FleftZ)) & " (m)"
  .Cells(RowNum + 7, 1) = "FrightZ = " & Trim(Str(FrightZ)) & " (m)"
  .Cells(RowNum + 8, 1) = "Rcoil = " & Trim(Str(Rcoil)) & " (Ohms)"
  .Cells(RowNum + 9, 1) = "Lcoil = " & Trim(Str(Lcoil)) & " (H)"
End With
' Write the output headers to the file.
With objExcelWS
  .Cells(RowNum + 10, 1) = "Z0 (m)"
  .Cells(RowNum + 10, 2) = "C (F)"
  .Cells(RowNum + 10, 3) = "Resr (O)"
  .Cells(RowNum + 10, 4) = "EC0 (J)"
  .Cells(RowNum + 10, 5) = "VC0 (V)"
  .Cells(RowNum + 10, 6) = "Imax (A)"

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```

        .Cells(RowNum + 10, 7) = "Send (m/s)"
        .Cells(RowNum + 10, 8) = "Eend (J)"
End With
RowNum = RowNum + 10
' Main Do loop.
' 1. Search # 1 is for a composite capacitor in which a
'   number of standard component capacitors are wired in
'   parallel. The standard component capacitor has
'   2200uF with Resr = 0.090 Ohms. Anywhere between
'   one and 20 component capacitors can be used.
' 2. Search #2 is for the best starting position. This
'   be anywhere in the range +/- 2 cm from the Z0mid
'   set above for each coil.
' 3. The maximum peak current allowed in 100 Amperes.
' 4. The maximum ending current, when the slug passes
'   through the center of the coil, is 0.5 Amperes.
'
Dim MaxPeakCurrent As Double = 100
Dim MaxEndingEnergy As Double = 0.01
Dim Z0midLastIteration As Double
Dim VC0startLastIteration As Double
Dim NumCapLastIteration As Double
Dim SendThisIteration As Double
Dim SendFurtherAway As Double
Dim SendCloserToFace As Double
' Set the opening values afor a coil.
NumCapLastIteration = NumCap
Z0midLastIteration = Z0mid
VC0startLastIteration = VC0start
Do
    ' Set the capacitance.
    C = 0.0022 * NumCapLastIteration
    Resr = 0.09 / NumCapLastIteration
    R = Resr + Rsw + Rcoil
    VC0 = VC0startLastIteration
    EC0 = 0.5 * C * VC0 * VC0
    QC0 = C * VC0
    Z0 = Z0midLastIteration
    ' Execute the run.
    IntegrateOneRun(False, objExcelWS, 0)
    SendThisIteration = Send
    ' Write the results to the file.
    RowNum = RowNum + 1
    With objExcelWS
        .Cells(RowNum, 1) = Z0
        .Cells(RowNum, 2) = C
        .Cells(RowNum, 3) = Resr
        .Cells(RowNum, 4) = EC0
        .Cells(RowNum, 5) = VC0
        .Cells(RowNum, 6) = MaxCurrent
        .Cells(RowNum, 7) = Send
        .Cells(RowNum, 8) = ECend + ELeend
    End With
    '////////////////////
    ' If the peak current is more than 0.01 A high or low, re-compute.
    If (Math.Abs(MaxCurrent - MaxPeakCurrent) > 0.01) Then
        VC0startLastIteration = VC0startLastIteration * _
            (1 + (MaxPeakCurrent - MaxCurrent) / MaxCurrent)
    End If
Loop

```

```

Else
'////////////////////
' Try a starting point further away from the face.
Z0 = Z0midLastIteration + 0.0005
IntegrateOneRun(False, objExcelWS, 0)
SendFurtherAway = Send
' Write the results to the file.
RowNum = RowNum + 1
With objExcelWS
    .Cells(RowNum, 1) = Z0
    .Cells(RowNum, 2) = C
    .Cells(RowNum, 3) = Resr
    .Cells(RowNum, 4) = EC0
    .Cells(RowNum, 5) = VC0
    .Cells(RowNum, 6) = MaxCurrent
    .Cells(RowNum, 7) = Send
    .Cells(RowNum, 8) = ECend + ELeud
End With
'////////////////////
' Try a starting point closer to the face.
Z0 = Z0midLastIteration - 0.0005
IntegrateOneRun(False, objExcelWS, 0)
SendCloserToFace = Send
' Write the results to the file.
RowNum = RowNum + 1
With objExcelWS
    .Cells(RowNum, 1) = Z0
    .Cells(RowNum, 2) = C
    .Cells(RowNum, 3) = Resr
    .Cells(RowNum, 4) = EC0
    .Cells(RowNum, 5) = VC0
    .Cells(RowNum, 6) = MaxCurrent
    .Cells(RowNum, 7) = Send
    .Cells(RowNum, 8) = ECend + ELeud
End With
' Move closer to the better starting position.
If ((SendThisIteration > SendFurtherAway) Or _
(SendThisIteration > SendCloserToFace)) Then
    If (SendThisIteration < SendFurtherAway) Then
        Z0midLastIteration = Z0midLastIteration - 0.00025
    Else
        Z0midLastIteration = Z0midLastIteration + 0.00025
    End If
Else
'////////////////////
' If the ending energy is too high, try a smaller capacitor.
' If the ending current is acceptable, then stop.
If ((ECend + ELeud) > MaxEndingEnergy * EC0) Then
    NumCapLastIteration = NumCapLastIteration - 1
Else
    Exit Do
End If
End If
End If
Loop
Next I
' Save and close the output file.
objExcelWB.Save()

```

```
objExcelWB.Close()  
objExcel.Quit()  
MsgBox("All done.")  
'////////////////////////////////////'  
'////////////////////////////////////'
```