Selecting barrel coils for an induction coil gun

Matching coils and capacitors in an induction gun is a delightful engineering problem. It's a classic case of whack-a-mole. Try to fix one thing and two others get further out of alignment. A simulation program like the one I described in the earlier paper titled *Using FEMM to design an induction coil gun* is helpful, but can mesmerize the user into trying to optimize by guessing. In this paper, I am going to try to work out some of the basic principles in the hope that they can eliminate some of the guesswork.

To put some bounds on the scope of the problem, I am going to focus my attention in the following ways:

1.   I am going to assume that each barrel coil circuit has only one capacitor.

2.   I am going to assume that these capacitors are the type used in defibrillator machines, and therefore have capacitances in the 32μF to 120μF range. These capacitors are polarized and cannot be charged up in the reverse direction.

3.   I am going to assume that each barrel coil circuit is controlled by an SCR, and that the SCR will terminate conduction of current if and when it turns negative.

4.   I am going to assume that the goal is to deliver kinetic energy to the armature in a single pulse.

Each of these assumptions does have an alternative, which may well give better performance. Even so, I will rely on these assumptions in this paper.

The onset of oscillation in an isolated barrel coil circuit

In this section, I will consider a single barrel coil, with nothing else present, not even the armature. The circuit is an L-R-C circuit like the following.
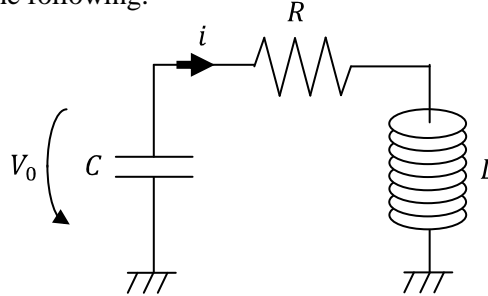
The differential equation which describes the circuit can be written in terms of the instantaneous current i as follows:

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 \quad (1)$$

It is customary to define time constants for the R-L combination $(\tau_{RL} = L/R)$ and for the R-C combination $(\tau_{RC} = RC)$ and to re-write the d.e. as:

$$\tau_{RL}\frac{d^2i}{dt^2} + \frac{di}{dt} + \frac{i}{\tau_{RC}} = 0 \quad (2)$$

The characteristic equation can be written down by inspection as:

$$\tau_{RL}\alpha^2 + \alpha + \frac{1}{\tau_{RC}} = 0 \quad (3)$$

This is a simple quadratic equation whose two roots can also be written down by inspection:

$$\alpha = \frac{-1 \pm \sqrt{1 - \frac{4\tau_{RL}}{\tau_{RC}}}}{2\tau_{RL}}$$

$$= -\frac{1}{2\tau_{RL}} \pm \frac{1}{2\tau_{RL}}\sqrt{1 - \frac{4\tau_{RL}}{\tau_{RC}}} \quad (4)$$

The behaviour of the current depends on the nature of the radical. If the radical is imaginary, the current will be an oscillating trigonometric function with a decaying amplitude. If the radical is real, the current's waveform will be the sum of two exponential terms, one with a rising amplitude and the second with a falling amplitude. The breakpoint between these two types of behaviour occurs when the radical is zero, namely:

$$1 = \frac{4\tau_{RL}}{\tau_{RC}}$$

$$\rightarrow \quad 1 = \frac{4L}{R^2C}$$

$$\rightarrow \quad L = \tfrac{1}{4}R^2C \quad (5)$$

The fourth assumption above requires that the capacitor deliver its energy in a single pulse. We do not want an oscillating solution. We want the radical to have a real value, so that:

$$1 > \frac{4\tau_{RL}}{\tau_{RC}}$$

$$\rightarrow \quad RC > \frac{4L}{R}$$

$$\rightarrow \quad L < \tfrac{1}{4}R^2C \quad (6)$$

For a given value of capacitance, we want a small value of inductance and, surprisingly, a large value of resistance. Those choices make it more likely that the inequality in Equation (6) will be satisfied.

Let's do a reality check. In the earlier paper, barrel coil #1 consisted of eight layers of AWG #16 enameled copper wire wound on a 2½ inch tube with 18 turns per layer. The resistance and self-inductance of the coil were found to be 0.3866Ω and $L = 1609.7\mu H$, respectively. Also included in the circuit was an SCR whose on-resistance was assumed to be 0.02Ω, so the total series resistance in the circuit was $R = 0.4066\Omega$. The capacitor in the circuit was $C = 32\mu F$. The inequality in Equation (6) was not satisfied.

$$1609.7\mu < \tfrac{1}{2} \times 0.4066^2 \times 32\mu$$

$$\rightarrow \quad 1609.7\mu < 2.65\mu$$

$$\text{NO}$$

In fact, it was unsatisfied by about three orders of magnitude. It is no surprise, therefore, that the currents we saw in the earlier paper were highly oscillatory.

One cure would be to use a bigger capacitor. The biggest capacitor we are allowed (by assumption #2) is 120μF. Using it moves us in the right direction, but not far enough.

Another cure would be use more resistance. That is true, but it would be a misguided cure. It would be perverse to willfully add resistance in order to drive the response into a single pulse. Burning off more energy as heat simply to avoid oscillations cannot be the right thing to do.

There is another cure: adding more layers to the coil. Adding more layers will increase both the inductance and the resistance. The resistance will increase at a faster rate than the inductance. Each additional layer adds less inductance than the previous layer since the degree of coupling is less, but it adds more resistance since the circumference is larger. Furthermore, the resistance term is squared in Equation (6), so we get more bang for our buck by taking this route.

Tentative conclusions:

1.   Use 120μF capacitors rather than 32μF capacitors.

2.   Wind more layers on the barrel coils.

Taking into account the effect of an armature fixed in position

In the previous section, the armature was completely ignored. In this section, I will bring the armature into the model. However, I will prevent it from moving. That means the mutual inductance M between the barrel coil and the armature will be constant. The appropriate schematic, with a couple of new symbols defined, is this.
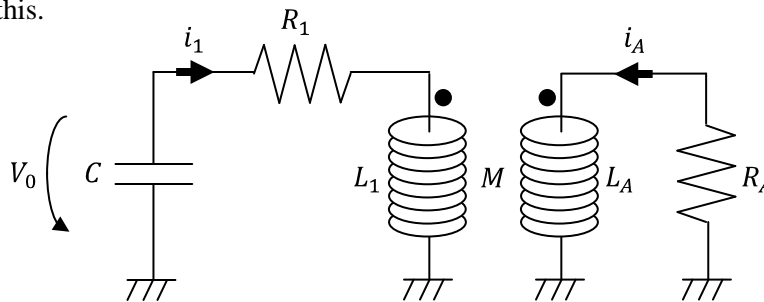
We can take over results directly from the earlier paper. The two circuit equations are:

$$\left. \begin{array}{ll} \text{Barrel coil:} & V_0 - \dfrac{1}{C}\int i_1 dt = R_1 i_1 + \dfrac{d}{dt}(L_1 i_1 + M i_A) \\[2mm] \text{Armature:} & \dfrac{d}{dt}(L_A i_A + M i_1) + R_A i_A = 0 \end{array} \right\} \quad (7)$$

Because the self-inductances and the mutual inductance are constant, they pass through the derivatives as constant. When we apply the product rule, we get:

$$\left. \begin{array}{ll} \text{Barrel coil:} & V_0 - \dfrac{1}{C}\int i_1 dt = R_1 i_1 + L_1 \dfrac{di_1}{dt} + M \dfrac{di_A}{dt} \\[2mm] \text{Armature:} & L_A \dfrac{di_A}{dt} + M \dfrac{di_1}{dt} + R_A i_A = 0 \end{array} \right\} \quad (8)$$

Taking the derivative of the first equation gives:

Barrel coil:
$$L_1 \frac{d^2 i_1}{dt^2} + M \frac{d^2 i_A}{dt^2} + R_1 \frac{di_1}{dt} + \frac{i_1}{C} = 0$$

Armature:
$$L_A \frac{di_A}{dt} + M \frac{di_1}{dt} + R_A i_A = 0$$

(9)

If we also take the derivative of the second equation and then substitute for the second derivative of i_A in the first equation, we get:

$$L_1 \frac{d^2 i_1}{dt^2} + M \left(-\frac{M}{L_A} \frac{d^2 i_1}{dt^2} - \frac{R_A}{L_A} \frac{di_A}{dt} \right) + R_1 \frac{di_1}{dt} + \frac{i_1}{C} = 0$$

$$\rightarrow \quad \left(L_1 - \frac{M^2}{L_A} \right) \frac{d^2 i_1}{dt^2} + R_1 \frac{di_1}{dt} - \frac{MR_A}{L_A} \frac{di_A}{dt} + \frac{i_1}{C} = 0 \qquad (10)$$

The solution will not be oscillatory if the coefficient of the second derivative term vanishes. <u>If that is the case</u>, then the governing differential equations become:

$$R_1 \frac{di_1}{dt} - \frac{MR_A}{L_A} \frac{di_A}{dt} + \frac{i_1}{C} = 0$$

$$L_A \frac{di_A}{dt} + M \frac{di_1}{dt} + R_A i_A = 0$$

(11)

This is such an important assumption that I will emphasize it as follows:

$$\text{Non-oscillatory solution} \quad \leftrightarrow \quad L_1 L_A = M^2 \qquad (12)$$

In this non-oscillatory case, let's assume that the solutions for i_1 and i_A have the following general exponential forms, where the coefficients and exponents may be complex.

$$i_1(t) = A e^{\alpha t}$$
$$i_A(t) = B e^{\beta t}$$

(13)

When we substitute these proposed solutions into the differential equations, we get:

$$A \left(R_1 \alpha + \frac{1}{C} \right) e^{\alpha t} - \frac{MR_A}{L_A} B\beta e^{\beta t} = 0$$

$$B(L_A \beta + R_A) e^{\beta t} + MA\alpha e^{\alpha t} = 0$$

(14)

We can divide both equations through by $e^{\alpha t}$ which, being an exponential term, will never be exactly zero. We get:

$$A \left(R_1 \alpha + \frac{1}{C} \right) - \frac{MR_A}{L_A} B\beta e^{(\beta - \alpha)t} = 0$$

$$B(L_A \beta + R_A) e^{(\beta - \alpha)t} + MA\alpha = 0$$

(15)

Take a look at the first equation in Equation (15). The first term is constant; the second term varies as time varies. The second equation has a similar structure. Now, if the forms of solution proposed in Equation (13) are really valid solutions, then they must satisfy the differential equations at all times.

Obviously, this can only happen if the time-dependent terms are somehow neutralized. There are only a couple of ways to neutralize them.

One way is to require that the coefficients of the time-dependent terms be zero and, separately, that one or more of the coefficients in the constant terms be zero. That way everything would zero itself out, as Equation (15) requires. Unfortunately, that does not work here. To suit the first equation, we would have to set $B = 0$. To suit the second equation, we would have to set $A = 0$. That makes the solution identically equal to zero, which is of no help.

Let's try something else. Perhaps we can find a way to make the exponential terms behave as constants, in which case we may be able to work something out with the constant terms to keep the whole equal to zero. There is a possibility. If we set the exponent $\beta - \alpha = 0$, then the exponent is zero for all values of time t. That means the exponential term is always equal to one, and Equation (15) reduces to:

$$\left. \begin{array}{l} A\left(R_1\alpha + \dfrac{1}{C}\right) - \dfrac{MR_A}{L_A}B\alpha = 0 \\[2mm] B(L_A\alpha + R_A) + MA\alpha = 0 \end{array} \right\} \quad (16)$$

We have two equations, but only one unknown α. Well, that's not quite true. The coefficients A and B are not yet known, either. Suppose we solve the second equation for B, in terms of A, and substitute that into the first equation. We get:

$$A\left(R_1\alpha + \dfrac{1}{C}\right) - \dfrac{MR_A}{L_A}\alpha\left(-\dfrac{MA\alpha}{L_A\alpha + R_A}\right) = 0$$

$$\rightarrow \quad \left(R_1\alpha + \dfrac{1}{C}\right) + \dfrac{R_A}{L_A}\left(\dfrac{M^2\alpha^2}{L_A\alpha + R_A}\right) = 0 \qquad (17)$$

Let's not forget the the governing assumption in Equation (12), that $M^2 = L_1L_A$. We can then re-arrange Equation (17) as:

$$R_1\alpha + \dfrac{1}{C} + \dfrac{R_A}{L_A}\left(\dfrac{L_1L_A\alpha^2}{L_A\alpha + R_A}\right) = 0$$

$$\rightarrow \quad \alpha + \dfrac{1}{R_1C} + \dfrac{L_1}{R_1}\left(\dfrac{\alpha^2}{\dfrac{L_A}{R_A}\alpha + 1}\right) = 0 \quad (18)$$

Now, I am going to talk about time constants. I've already defined time constants $\tau_{R_1C} = R_1C$ and $\tau_{R_1L_1} = L_1/R_1$ for the barrel coil circuit (with subscripts added to identify the specific components now that there are two coils and two resistors). At this point, I am going to introduce a time constant for the armature circuit. The armature circuit does not have a capacitor. It therefore has only one time constant, of the R-L variety. I am going to use the symbol $\tau_A = L_A/R_A$ to represent that time constant. We can now re-state Equation (18) entirely in terms of time constants:

$$\alpha + \dfrac{1}{\tau_{R_1C}} + \tau_{R_1L_1}\left(\dfrac{\alpha^2}{\tau_A\alpha + 1}\right)$$

$$\rightarrow \quad \tau_{R_1C}\left(\tau_A + \tau_{R_1L_1}\right)\alpha^2 + \left(\tau_A + \tau_{R_1C}\right)\alpha + 1 = 0 \quad (19)$$

There will be two roots for exponent α.

$$\alpha = \frac{-\left(\tau_A + \tau_{R_1 C}\right) \pm \sqrt{\left(\tau_A + \tau_{R_1 C}\right)^2 - 4\tau_{R_1 C}\left(\tau_A + \tau_{R_1 L_1}\right)\left(\tau_A + \tau_{R_1 C}\right)}}{2\tau_{R_1 C}\left(\tau_A + \tau_{R_1 L_1}\right)} \qquad (20)$$

In principle, we cannot allow the argument of the square root to be negative. If it is negative, then the exponent α will contain an imaginary component, which represents oscillation and contradicts the governing assumption that the solution not be oscillatory. We can write the positivity requirement as follows:

$$\left(\tau_A + \tau_{R_1 C}\right)^2 > 4\tau_{R_1 C}\left(\tau_A + \tau_{R_1 L_1}\right)\left(\tau_A + \tau_{R_1 C}\right)$$
$$\rightarrow \qquad \left(\tau_A + \tau_{R_1 C}\right) > 4\tau_{R_1 C}\left(\tau_A + \tau_{R_1 L_1}\right)$$
$$\rightarrow \qquad 1 + \frac{\tau_A}{\tau_{R_1 C}} > 4\left(\tau_A + \tau_{R_1 L_1}\right) \qquad (21)$$

This is the condition that ensures no oscillations. Things one can do to encourage this condition are:

1. Reduce the time constant of the barrel coil's R_1-C pair. Reducing $\tau_{R_1 C}$ will tend to increase the left hand side. Since $\tau_{R_1 C} = R_1 C$, reducing the time constant can be achieved by reducing the resistance or reducing the capacitance, or both. Note that both of these changes are exactly opposite to the tentative conclusions reached in the previous section.

2. Reduce the armature's time constant or the barrel coil's R_1-L_1 time constant, or both. This will reduce the value of the right-hand side.

The condition $L_1 L_A = M^2$ is impossible

The non-oscillatory condition $L_1 L_A = M^2$ in Equation (12) is physically impossible. The real question for us is: can we get close enough to this ideal to even pursue the goal of single-pulse energy delivery.

A coupling coefficient k is defined for the interaction of two coils, such that:

$$k = \frac{M}{\sqrt{L_1 L_A}} \qquad (22)$$

The non-oscillatory condition therefore boils down to $k = 1$. Ideal transformers, with absolutely no leakage, have $k = 1$. From the point-of-view of the primary circuit, an ideal transformer "disappears" as a discrete component and can be replaced by a scaled version of the "load" in the secondary circuit. The secondary circuit in our case is the armature. Its "load" is the armature resistance. If $k = 1$, this load will be seen by the primary circuit as a pure resistance.

Two air-cored coils, one inside the other, will never interact with $k = 1$. There will always be some leakage and that leakage will cause there to be some degree of oscillation in the primary (barrel coil) circuit. Let me use the values from the earlier paper to illustrate how far away from the ideal we are.

The armature in the earlier paper consisted of two layers of AWG #10 enameled copper wire, with nine turns per layer, and an outside diameter chosen so the armature would just fit inside the barrel tube. The

resistance and self-inductance of the armature were found to be $R_A = 0.00943\Omega$ and $L_A = 17.44\mu H$, respectively.

I have prepared a simple LTSpice model (shown here) to simulate the result of assuming that $k = 1$.

```
.tran 0 0.01 0 startup
.ic V(V0)=5000
K1 L1 LA 1
```
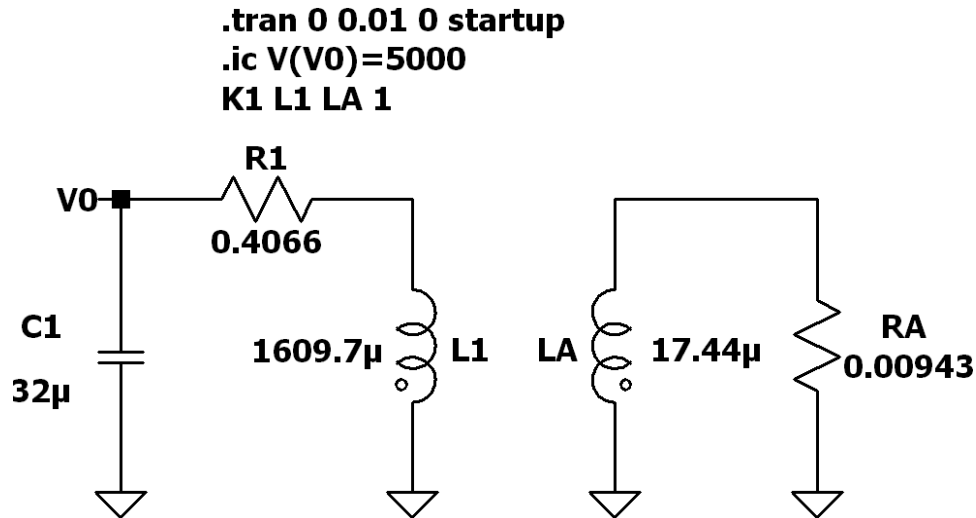
This schematic has the same component values that were used in the simulation in the previous paper, except for the mutual inductance. Power is powered by a 32μF capacitor initially charged up to 5000V. The coupling coefficient is set to one, so that $M = \sqrt{L_1 L_A}$. The following graph shows what happens with this perfect coupling. It shows the currents flowing through the barrel coil (in black) and the armature (in red). It also shows the voltage drop over the capacitor (in green).
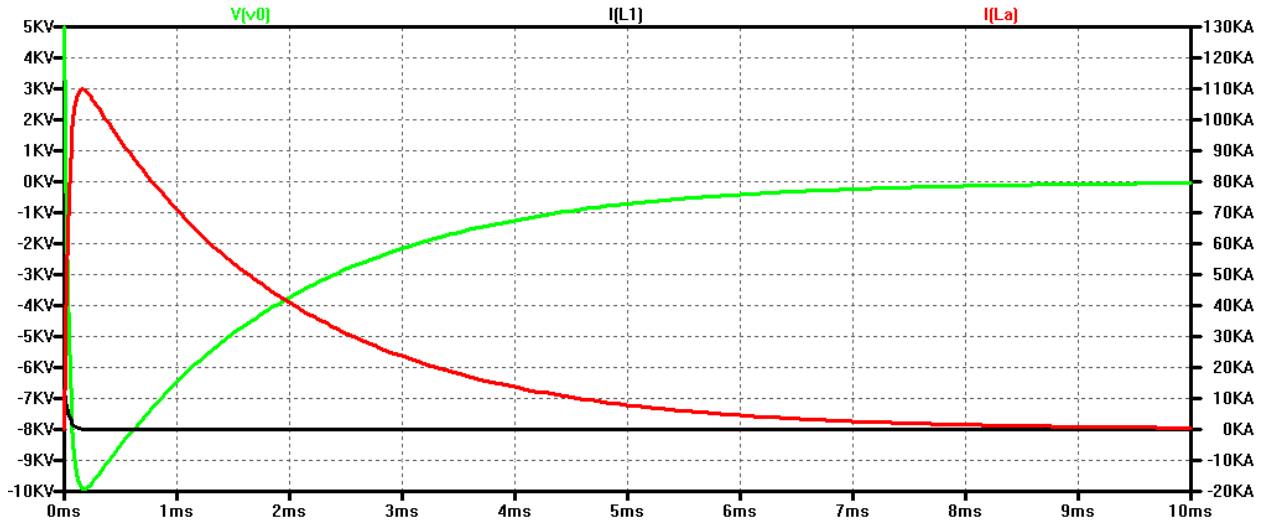
The armature current is a nice single pulse. The barrel coil current is nice, too. It declines steadily from 10KA to zero, with no overshoot. Not so nice is the voltage over the capacitor. The capacitor discharges very quickly, and then continues to charge up in the reverse direction, reaching twice the reverse voltage that it had originally. This capacitor would have blown itself up after a few microseconds. But, let's put that issue aside for now.

If the barrel coil and armature were perfectly coupled, their mutual inductance would be $M = \sqrt{L_1 L_A} = \sqrt{1609.7\mu \times 17.44\mu} = 167.55\mu H$. During the simulation in the earlier paper, the mutual inductance

never got that high. For much of the simulation, the mutual inductance was closer to 93μH. (The mutual inductance was not constant because the armature was moving.) This corresponds to a coupling coefficient $k = 93\mu/167.55\mu = 0.555$. Re-running the LTSpice simulation of the above circuit with $k = 0.555$ gives the following waveforms.
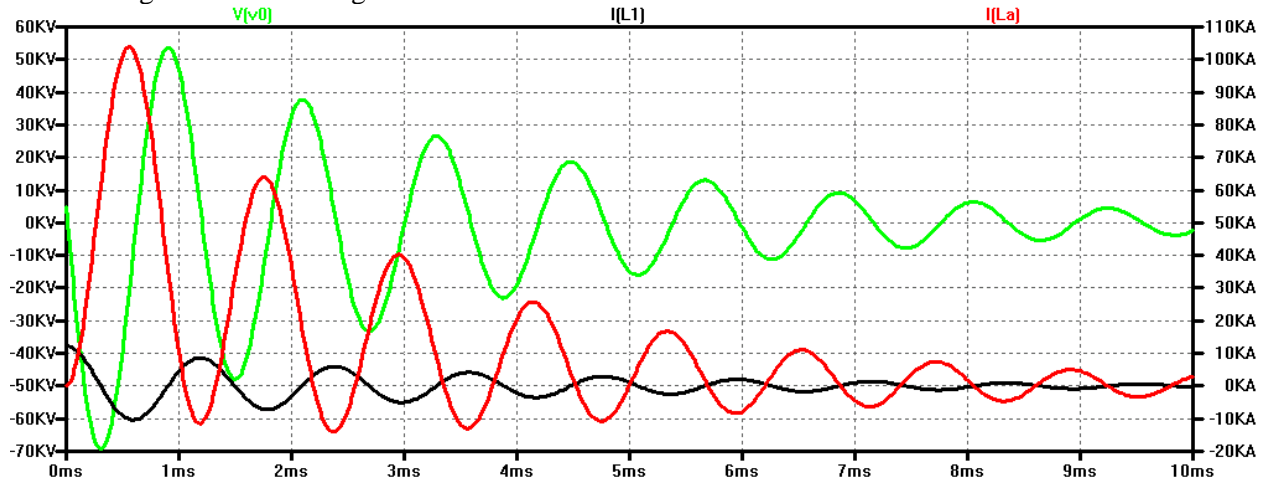
Not only is the energy not delivered in a single pulse, but the voltage to which the capacitor is subjected is simply outrageous.

Revisiting the parameters of the barrel coil circuit

It is possible that the source of the problem has more to do with the barrel coil than the armature. It could be that the barrel coil was so badly chosen that it was unrealistic to expect that the armature could cope with it. Let's start over, and see if it is possible to design a barrel coil which has less of a tendency to oscillate. The restrictions I am going to impose are:

1.   The coil's length must be as close to one inch as possible, to match the length of the armature.

2.   The coil's inner diameter must be 2½ inches, as before, to suit the tube size.

3.   The circuit will still be controlled by an SCR with an on-resistance of 20mΩ.

4.   The capacitor will remain at 32μF.

The only things we can vary are the number of layers of wire in the winding and the size of wire used. I am going to prepare a Lua script to loop through some combinations of layers and wire sizes. The figure of merit I will use to compare the outcomes is the ratio of time constants. Equation (6) above was the condition which determines when the barrel coil, in isolation, will or will not oscillate. Equation (6) can be re-stated in terms of time constants as follows. The magic ratio is 0.25. If the inductive time constant is less than one quarter of the capacitive time constant, the barrel coil by itself will not oscillate.

$$(6): \quad L_1 < \tfrac{1}{4}R_1^2 C$$
$$\rightarrow \quad \left(\frac{L_1}{R_1}\right)\left(\frac{1}{R_1 C}\right) < \tfrac{1}{4}$$
$$\rightarrow \quad \frac{\tau_{R_1 L_1}}{\tau_{R_1 C}} < \tfrac{1}{4} \qquad (23)$$

I have attached as Appendix "A" a copy of the Lua script I used to prepare the data for the following two graphs. The graphs show the time constant ratio on the left-hand side of Equation (23) with respect to

~ 8 ~

the number of layers in the winding (along the horizontal), with different curves for different wire gauges. There were too many wire gauges to show all in one graph, so I used two. I have put a black dot on the spot which characterizes barrel coil #1 from the earlier paper, with eight layers of AWG #16 wire.

Barrel coil -- ratio of time constants

Value in earlier paper
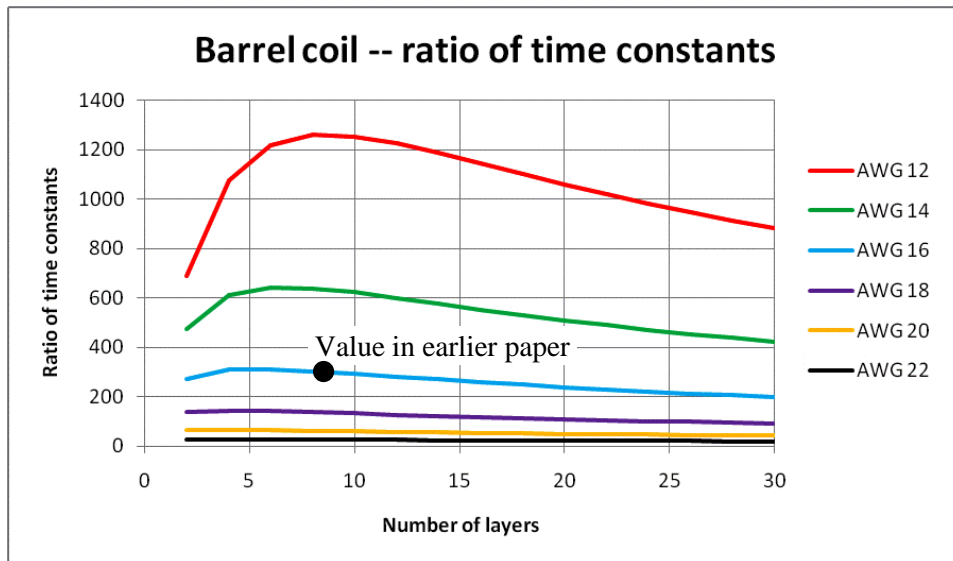
Barrel coil #1 used in the earlier paper has a time constant ratio of about 300 – it is no surprise to us now that the circuit oscillated.

It is clear from the graph that the size of the wire (the different curves) has a much bigger impact on the ratio of time constants than does the number of layers (along the horizontal). In fact, for smaller wire diameters, the ratio of time constants looks to be almost independent of the number of layers. It is also obvious that the time constant ratios for the wire sizes shown in the graph above are far above the desired limit. In order to get anywhere close to a time constant ratio of 0.25, smaller diameter wire must be used. The graph is continued below for smaller wire sizes.

Barrel coil -- ratio of time constants (cont'd)

Value in following simulation
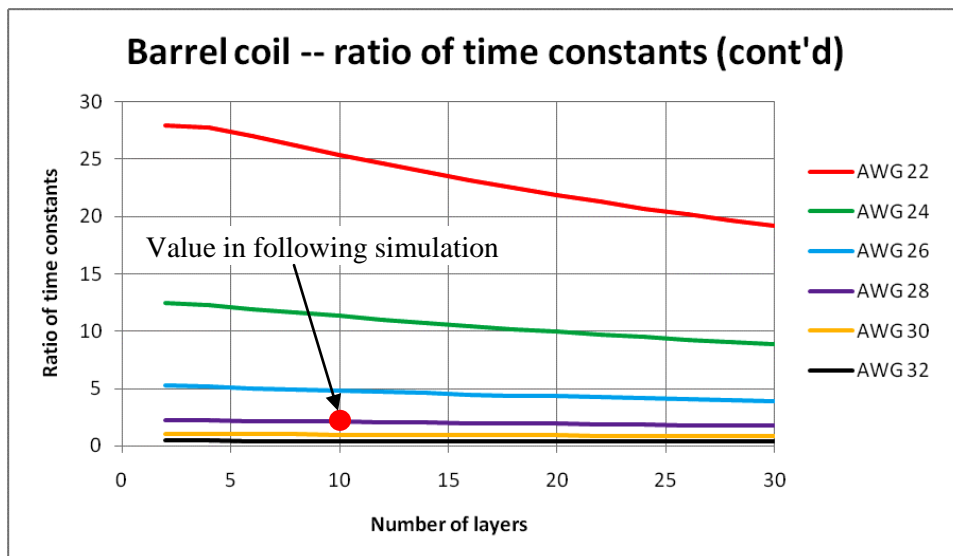
Equation (23) is not the only determinant of oscillatory behaviour, so the 0.25 value is not the end of the story, but it does look as if we are going to be stuck using wire gauges AWG #26 or higher.

There are other issues we cannot ignore. The ratio of the time constants is certainly important, but so too are the time constants themselves. A time constant is a measure of how quickly a circuit responds. The following graph shows the value of the R-L time constant $\tau_{R_1 L_1}$ for the four most suitable wire gauges. The horizontal axis is, once again, the number of layers. The vertical axis, along which the time constants are measured, is labeled in seconds.
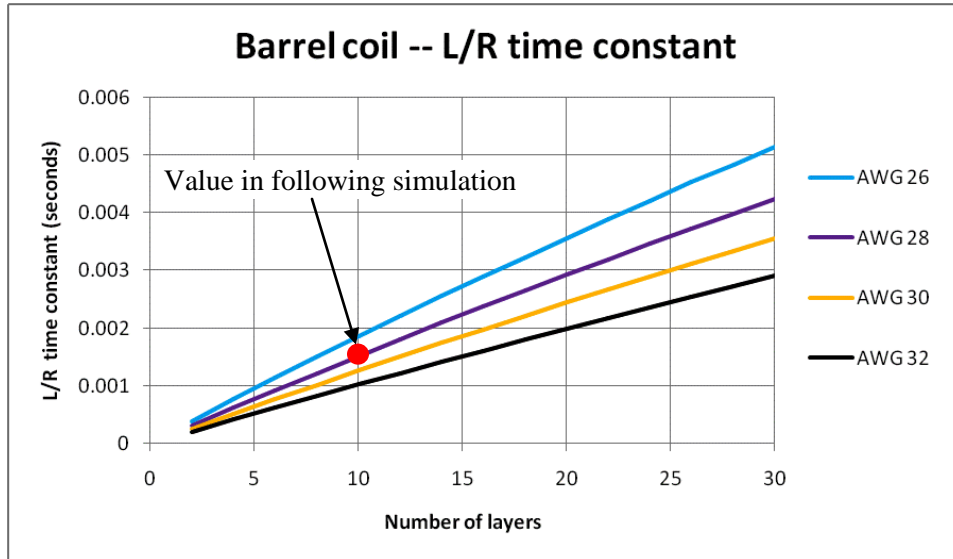
The issue which is going to make this graph important is the speed of the armature. Since the armature will be starting up from rest when it is under the influence of barrel coil #1, it will spend more time in and near barrel coil #1 than any of the other barrel coils further down the barrel. As the armature picks up speed, it will spend less and less time under the influence of each successive barrel coil.

The armature will probably remain under the influence of barrel coil #1 for no more than about three milliseconds, or 0.003 seconds. (If it spends more time, it is not going very fast!) In fact, if we are successful in designing the gun, the armature will spend only a few dozen microseconds under the influence of later coils. This requirement is going to put an upper limit on how many layers we can put in the barrel coils.

<u>**First numerical example**</u>

At this point, I am going to re-group, and run another time simulation using a new barrel coil. I will choose the properties of the barrel coil using the conclusions from the foregoing analysis. I will use ten layers of AWG #28 wire. The red dot in the previous two graphs marks this choice. For this configuration, the ratio of time constants and the R-L time constant are:

$$\left. \begin{array}{r} \dfrac{\tau_{R_1 L_1}}{\tau_{R_1 C}} = 2.1 \\[2mm] \tau_{R_1 L_1} = 1.52 \text{ ms} \end{array} \right\} \quad (24)$$

Notice that I did not take the ratio of time constants all the way down to 0.25. I stopped at 2.1 to leave a bit of oscillation so that the effects of other things can be observed later. The circuit resistance and self-inductance of barrel coil #1 which produce these time constants are:

$$\left. \begin{array}{r} R_1 = 22.7\Omega \\ L_1 = 34{,}400\mu\text{H} \end{array} \right\} \quad (25)$$

Both the resistance and self-inductance are much higher than they were in the earlier paper, where they were 0.4066Ω and $1609.7\mu H$, respectively.

For this time simulation, I will leave everything else unchanged, including the armature (two layers of AWG #10 wire), the capacitance ($32\mu F$), its starting volatage (5000V) and the armature's starting location (protruding one-half inch through the muzzle end of barrel coil #1). I have made two administrative changes as well: (i) barrel coil #2 is disabled in this simulation, which therefore shows the effects of barrel coil #1 only, and (ii) the SCR in barrel coil #1's circuit is disabled, so the current can become negative if it wants to. The following graphs show the results. The first graph shows the current in barrel coil #1 (in black) and the current in the armature (in red). It looks like the oscillation has been reduced to a single cycle, but not quite a single positive pulse. The current through barrel coil #1 does wander slightly into negative territory, but not nearly so much as before.
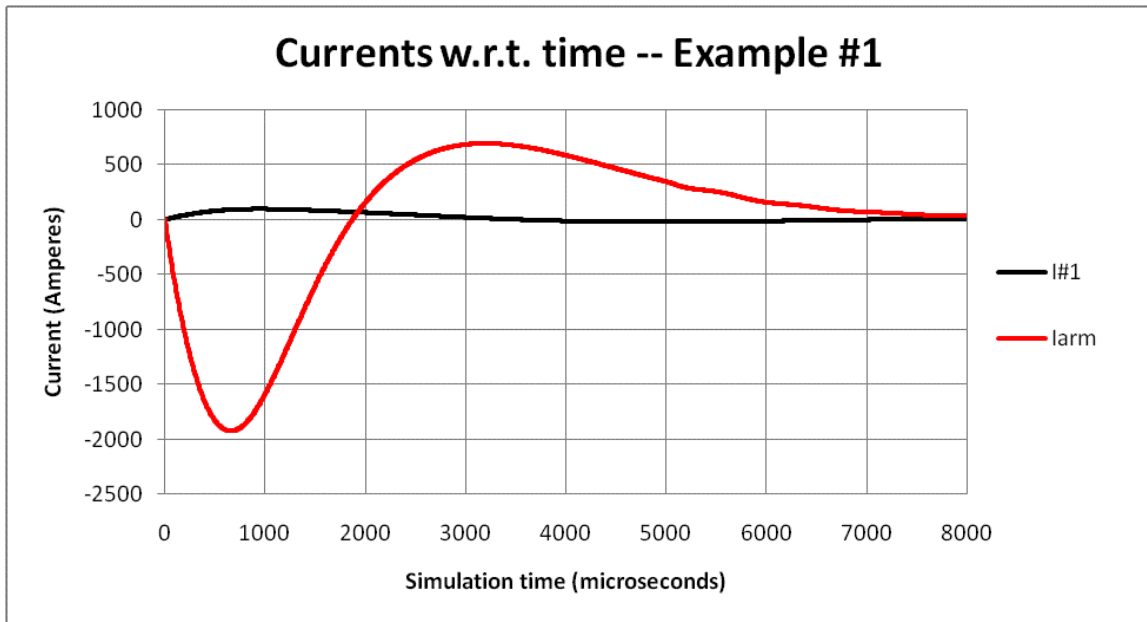
The next graph shows the voltage drop over the capacitor (in black) and the force exerted on the armature (in blue). The voltage, too, is much better than before, but still not good enough to prevent the capacitor's destruction. Note that there is a period, from about two to three milliseconds, when the force on the armature is negative, slowing it down.
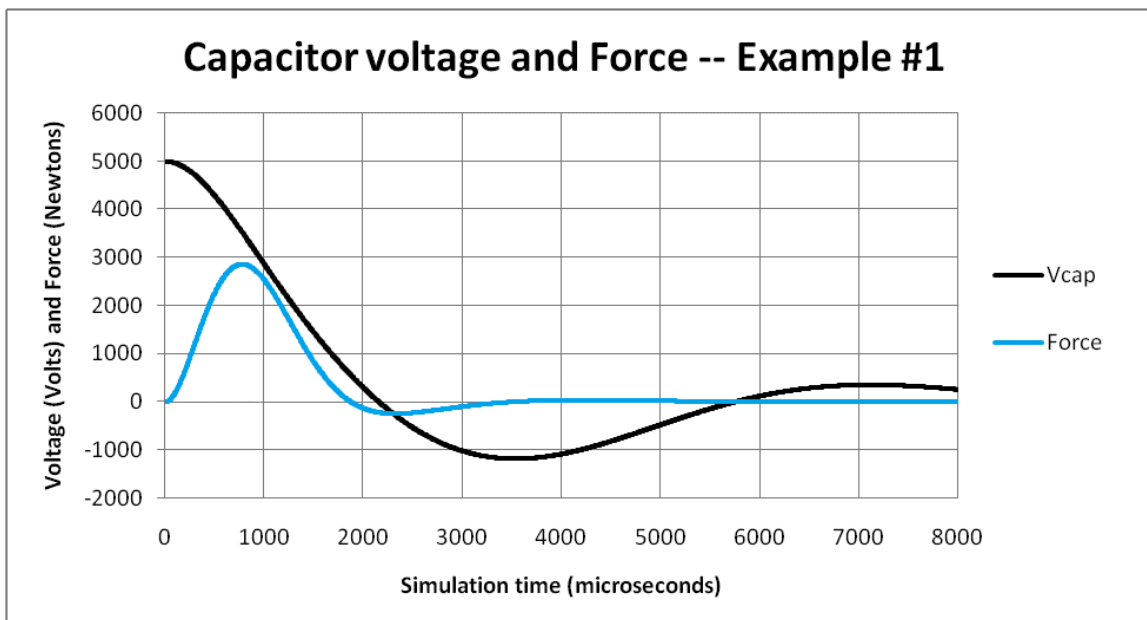
Second numerical example

One of the conclusions we reached above is that a bigger capacitor is better. This second numerical example is a repeat of the first numerical example, with the capacitance increased from 32μF to 120μF. In order to make the change more realistic, I have reduced the initial voltage from 5000 Volts to 2580 Volts. This ensures that both examples start out with the same amount of stored energy – 400 Joules. This change in capacitance will not change the R-L time constant from the first example, but it will increase the R-C time constant by a factor of about four. Taken together, the change will reduce the time constant ratio by a factor of about four, from 2.1 to about one-half. The new values are:

$$\left.\begin{array}{c} \tau_{R_1 L_1} = \dfrac{L}{R_1} = \dfrac{34{,}400\mu\text{H}}{22.7\Omega} = 0.00152 \text{ sec} \\[2mm] \tau_{R_1 C} = R_1 C = 22.7\Omega \times 120\mu\text{F} = 0.00272 \text{ sec} \\[2mm] \dfrac{\tau_{R_1 L_1}}{\tau_{R_1 C}} = \dfrac{0.00152}{0.00272} = 0.56 \end{array}\right\} \quad (26)$$

The following two graphs show the results for this example, in the same format as the first one.
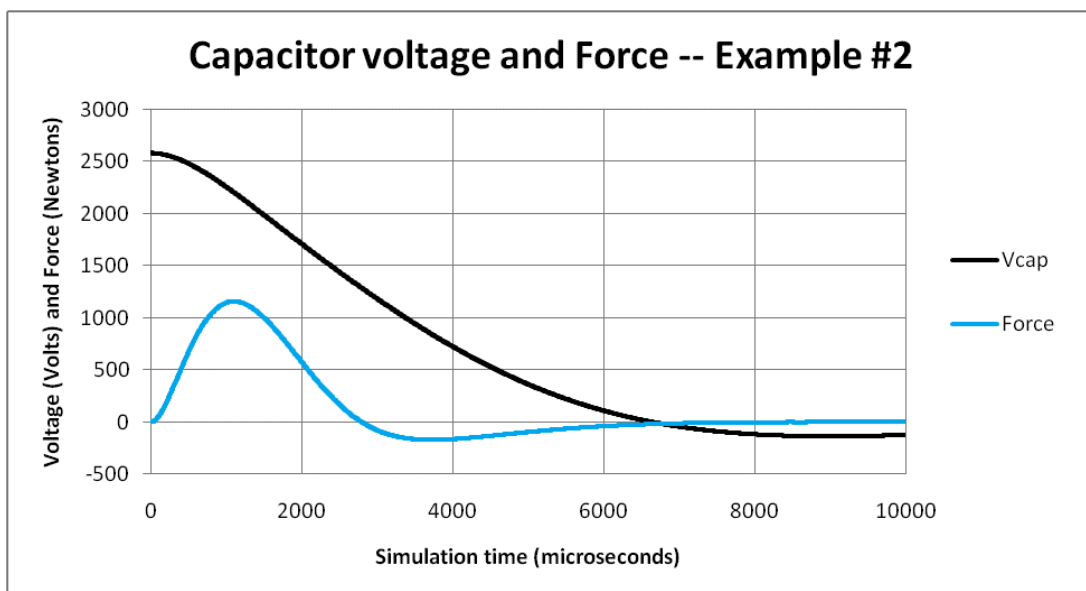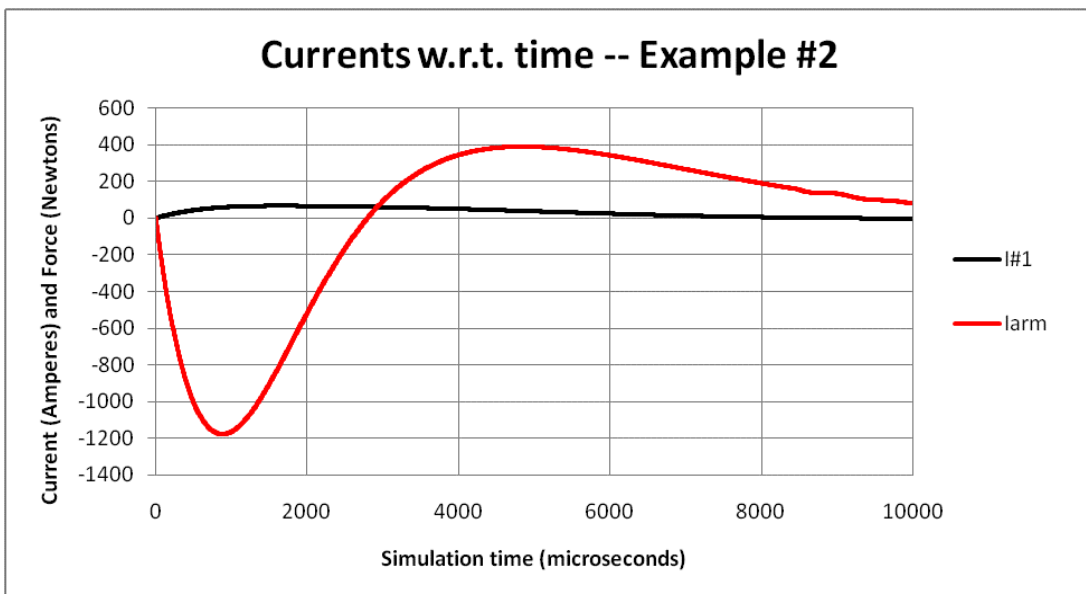
Because the time constant ratio is still greater than 0.25, we would expect to see, and do see, a bit of oscillation remaining. Both the barrel coil's current and the capacitor's voltage make only minor excursions into negative territory.

Terminating this line of analysis

I have decided to end this line of analysis. In this paper, I have been trying to choose the values of the components in a barrel coil circuit in such a way that the response is not oscillatory. The basic problem is the tendency of energy to cycle back and forth between the capacitor and the inductor.

It occurs to me that there is another way to stop current from carrying energy back to the capacitor. Diodes permit current to flow only one way. A diode placed in series with the capacitor would not be enough, since it would not prevent the capacitor from charging up in the reverse direction during the first half-cycle of operation. But a diode placed in parallel across the capacitor could bypass current around the capacitor and prevent it from reverse-charging

I am going to examine this in the next paper.

Jim Hawley
© June 2015

If you found this description helpful, please let me know. If you spot any errors or omissions, please send an e-mail. Thank you.

Listing of Lua script to calculate time constant ratios

```lua
-- Important variables related to the simulation process
MaxMeshSize = 0.01        -- Maximum FEMM triangle size
MaxResidual = 1E-8        -- Maximum FEMM residual for convergence test

-- Definition of Groups
-- Group(9) is the air
-- Group(1) is barrel coil #1

-- ALL DIMENSIONS ARE IN INCHES UNLESS A SUFFIX STATES OTHERWISE

-- Specify target length for the coil.  The number of turns will be calculated.
BCoilTargetLength = 1

-- Specify the z-location of the negative end of the coil.
BCoilNegEndZ = 0

-- Specify the inner and outer diameters of the barrel tube
TubeOuterDiam = 2.5
TubeOuterRad = TubeOuterDiam / 2

-- Specify the capacitance
Capacitance = 0.000032

-- Specify the dimensions of the bounding air spheres
AirLocalRad = 8
AirExtDiam = 1

--////////////////////////////////////////////////////////////////////////////
--// Open a text file for output and write a short header
--////////////////////////////////////////////////////////////////////////////
handle = openfile("./Barrel_coil_parameters.txt","a")
write(handle,"\nBarrel coil parameters:\n")
write(handle,"  FEMM maximum mesh size = ",MaxMeshSize,"\n")
write(handle,"  FEMM maximum residual = ",MaxResidual,"\n")
write(handle,"  Capacitance = ",Capacitance,"\n")

--////////////////////////////////////////////////////////////////////////////
--// This is the main loop through the ranges of parameters
--////////////////////////////////////////////////////////////////////////////
for AWGGauge = 12,32 do
    for NumLayers = 2,30,2 do

        newdocument(0)                              -- New magnetics problem
        mi_probdef(0,'inches','axi',MaxResidual,0,30)   -- Axisymmetric, in inches
        mi_grid_snap('off')                         -- Do not snap to a grid

        -- Set the parameters of the enameled copper wire used to wind the coil
        if (AWGGauge == 1) then
            WireExtDiamMM = 7.496
            WireBareDiamMM = 7.348
        end
```

```
if (AWGGauge == 2) then
    WireExtDiamMM = 6.690
    WireBareDiamMM = 6.543
end
if (AWGGauge == 3) then
    WireExtDiamMM = 5.971
    WireBareDiamMM = 5.827
end
if (AWGGauge == 4) then
    WireExtDiamMM = 5.330
    WireBareDiamMM = 5.189
end
if (AWGGauge == 5) then
    WireExtDiamMM = 4.757
    WireBareDiamMM = 4.620
end
if (AWGGauge == 6) then
    WireExtDiamMM = 4.246
    WireBareDiamMM = 4.115
end
if (AWGGauge == 7) then
    WireExtDiamMM = 3.790
    WireBareDiamMM = 3.665
end
if (AWGGauge == 8) then
    WireExtDiamMM = 3.383
    WireBareDiamMM = 3.264
end
if (AWGGauge == 9) then
    WireExtDiamMM = 3.023
    WireBareDiamMM = 2.906
end
if (AWGGauge == 10) then
    WireExtDiamMM = 2.703
    WireBareDiamMM = 2.588
end
if (AWGGauge == 11) then
    WireExtDiamMM = 2.418
    WireBareDiamMM = 2.304
end
if (AWGGauge == 12) then
    WireExtDiamMM = 2.163
    WireBareDiamMM = 2.052
end
if (AWGGauge == 13) then
    WireExtDiamMM = 1.934
    WireBareDiamMM = 1.829
end
if (AWGGauge == 14) then
    WireExtDiamMM = 1.732
    WireBareDiamMM = 1.628
end
if (AWGGauge == 15) then
    WireExtDiamMM = 1.549
    WireBareDiamMM = 1.450
```

```
end
if (AWGGauge == 16) then
    WireExtDiamMM = 1.384
    WireBareDiamMM = 1.290
end
if (AWGGauge == 17) then
    WireExtDiamMM = 1.240
    WireBareDiamMM = 1.151
end
if (AWGGauge == 18) then
    WireExtDiamMM = 1.110
    WireBareDiamMM = 1.024
end
if (AWGGauge == 19) then
    WireExtDiamMM = 0.993
    WireBareDiamMM = 0.912
end
if (AWGGauge == 20) then
    WireExtDiamMM = 0.892
    WireBareDiamMM = 0.813
end
if (AWGGauge == 21) then
    WireExtDiamMM = 0.800
    WireBareDiamMM = 0.724
end
if (AWGGauge == 22) then
    WireExtDiamMM = 0.714
    WireBareDiamMM = 0.643
end
if (AWGGauge == 23) then
    WireExtDiamMM = 0.643
    WireBareDiamMM = 0.574
end
if (AWGGauge == 24) then
    WireExtDiamMM = 0.577
    WireBareDiamMM = 0.511
end
if (AWGGauge == 25) then
    WireExtDiamMM = 0.516
    WireBareDiamMM = 0.455
end
if (AWGGauge == 26) then
    WireExtDiamMM = 0.462
    WireBareDiamMM = 0.404
end
if (AWGGauge == 27) then
    WireExtDiamMM = 0.419
    WireBareDiamMM = 0.361
end
if (AWGGauge == 28) then
    WireExtDiamMM = 0.373
    WireBareDiamMM = 0.320
end
if (AWGGauge == 29) then
    WireExtDiamMM = 0.338
```

```
        WireBareDiamMM = 0.287
end
if (AWGGauge == 30) then
    WireExtDiamMM = 0.307
    WireBareDiamMM = 0.254
end
if (AWGGauge == 31) then
    WireExtDiamMM = 0.275
    WireBareDiamMM = 0.226
end
if (AWGGauge == 32) then
    WireExtDiamMM = 0.247
    WireBareDiamMM = 0.203
end
BCoilWireExtDiamMM = WireExtDiamMM
BCoilWireBareDiamMM = WireBareDiamMM
BCoilWireExtDiam = BCoilWireExtDiamMM / 25.4
BCoilWireBareDiam = BCoilWireBareDiamMM / 25.4

-- Calculate the length of the coil.  This is the integral number of
-- turns of the wire used which best approximates the target length.
BCoilNumTurns = floor(0.5 + (BCoilTargetLength / BCoilWireExtDiam))
BCoilActualLength = BCoilNumTurns * BCoilWireExtDiam

-- Define all nodes for the surrounding air, in Group(9)
mi_addnode(0,AirLocalRad)                          -- Top of local sphere
mi_addnode(0,-AirLocalRad)                         -- Bottom of local sphere
mi_addnode(0,AirLocalRad + AirExtDiam)             -- Top of external sphere
mi_clearselected()
mi_selectnode(0,AirLocalRad)
mi_selectnode(0,-AirLocalRad)
mi_selectnode(0,AirLocalRad + AirExtDiam)
mi_setnodeprop('',9)

-- Clarify the (r,z) co-ordinates of the four corners of the windings of
-- the barrel coil.  It is wound directly on the tube's outer surface.
BCoilInnerRad = TubeOuterRad
BCoilOuterRad = BCoilInnerRad + (NumLayers * BCoilWireExtDiam)
BCoilPosEndZ = BCoilNegEndZ + BCoilActualLength

-- Define all nodes for the barrel coil.
mi_addnode(BCoilInnerRad,BCoilPosEndZ)             -- Top inside corner
mi_addnode(BCoilOuterRad,BCoilPosEndZ)             -- Top outside corner
mi_addnode(BCoilOuterRad,BCoilNegEndZ)             -- Bottom outside corner
mi_addnode(BCoilInnerRad,BCoilNegEndZ)             -- Bottom inside corner
mi_clearselected()
mi_selectnode(BCoilInnerRad,BCoilPosEndZ)
mi_selectnode(BCoilOuterRad,BCoilPosEndZ)
mi_selectnode(BCoilOuterRad,BCoilNegEndZ)
mi_selectnode(BCoilInnerRad,BCoilNegEndZ)
mi_setnodeprop('',1)

-- Define all line segments for the surrounding air, in Group(9)
-- There are two line segments:
-- 1. From the bottom of the local sphere to the top of the local sphere
```

```
-- 2. The diameter line across the external sphere
mi_addsegment(0,-AirLocalRad,0,AirLocalRad)
mi_addsegment(0,AirLocalRad,0,AirLocalRad+AirExtDiam)
mi_clearselected()
mi_selectsegment(0,AirLocalRad-0.1)
mi_selectsegment(0,AirLocalRad+0.1)
mi_setsegmentprop('',0,1,0,9)

-- Define a periodic boundary condition
mi_addboundprop('PeriodicBC',0,0,0,0,0,0,0,0,4)

-- Define all arcs for the surrounding air, in Group(0)
-- There are two arcs:
-- 1. Enclosing the local sphere
-- 2. Enclosing the external sphere
mi_addarc(0,-AirLocalRad,0,AirLocalRad,180,1)
mi_addarc(0,AirLocalRad,0,AirLocalRad+AirExtDiam,180,1)
mi_clearselected()
mi_selectarcsegment(AirLocalRad,0)
mi_selectarcsegment(AirExtDiam/2,AirLocalRad+(AirExtDiam/2))
mi_setarcsegmentprop(1,'PeriodicBC',0,9)

-- Define all line segments for the barrel coil
mi_addsegment(BCoilInnerRad,BCoilPosEndZ,BCoilOuterRad,BCoilPosEndZ)
mi_addsegment(BCoilOuterRad,BCoilPosEndZ,BCoilOuterRad,BCoilNegEndZ)
mi_addsegment(BCoilOuterRad,BCoilNegEndZ,BCoilInnerRad,BCoilNegEndZ)
mi_addsegment(BCoilInnerRad,BCoilNegEndZ,BCoilInnerRad,BCoilPosEndZ)
mi_clearselected()
mi_selectsegment(BCoilInnerRad+0.001,BCoilPosEndZ)
mi_selectsegment(BCoilOuterRad,BCoilPosEndZ-0.001)
mi_selectsegment(BCoilOuterRad-0.001,BCoilNegEndZ)
mi_selectsegment(BCoilInnerRad,BCoilNegEndZ+0.001)
mi_setsegmentprop('',0,1,0,I)

-- Define a block label for the air, in Group(9)
CentroidRLocal = 0.25
CentroidZLocal = AirLocalRad - 0.25
mi_addblocklabel(CentroidRLocal,CentroidZLocal)
CentroidRExt = 0.25
CentroidZExt = AirLocalRad + (AirExtDiam/2)
mi_addblocklabel(CentroidRExt,CentroidZExt)
mi_clearselected()
mi_selectlabel(CentroidRLocal,CentroidZLocal)
mi_selectlabel(CentroidRExt,CentroidZExt)
mi_getmaterial('Air')
mi_setblockprop('Air',0,0,0,0,9,0)

-- Describe the external region as a Kelvin transformation
mi_defineouterspace(AirLocalRad+(AirExtDiam/2),AirExtDiam/2,AirLocalRad)

-- Define new materials for the enameled copper wire being used
-- Relative permeability in r-direction = 1
-- Relative permeability in z-direction = 1
-- Permanent magnet coercivity = 0
-- Applied source current density = 0
```

```
-- Electrical conductivity = 58 MS/m for copper wire
-- Lamination thickness = 0
-- Hysteresis lag angle = 0
-- Lamination fill fraction = 1
-- Lamination type = 3 (This code identifies magnet wire)
-- Hysteresis lag angle in the x-direction = 0
-- Hysteresis lag angle in the y-direction = 0
-- Number of strands in wire = 1
-- Diameter of wire (in millimeters) has already been specified above
mi_addmaterial("BCoilWire",1,1,0,0,58,0,0,1,3,0,0,1,BCoilWireExtDiamMM)

-- Define block labels for the barrel coil.
Current = 100
CentroidR = (BCoilInnerRad + BCoilOuterRad) / 2
CentroidZ = (BCoilPosEndZ + BCoilNegEndZ) / 2
mi_addcircprop("BCoil",Current,1)
mi_addblocklabel(CentroidR,CentroidZ)
mi_clearselected()
mi_selectlabel(CentroidR,CentroidZ)
TotalNumTurns = NumLayers * BCoilNumTurns
mi_setblockprop("BCoilWire",0,MaxMeshSize,"BCoil",0,1,TotalNumTurns)

--////////////////////////////////////////////////////////////////////////
--// Now that the geometry has been specified, we can save the construction
--// in a temporary file which will be a sister file to this Lua script.
--// Then, we can get ready to do some analysis.
--////////////////////////////////////////////////////////////////////////
mi_saveas("./temp.fem")         -- Save the geometry
main_maximize()                 -- Maximize the main FEMM window
mi_zoomnatural()                -- Zoom the display for the best fit
showconsole()                   -- Show the Lua output window
clearconsole()                  -- Clear the Lua output window
mi_seteditmode("group")         -- Make edits such as translation by Group

--////////////////////////////////////////////////////////////////////////
--// Analyze the barrel coil
--////////////////////////////////////////////////////////////////////////
mi_analyze()
mi_loadsolution()
-- val1=current, val2=voltage and val3=flux
val1,val2,val3 = mo_getcircuitproperties("BCoil")
BCoilRfemm = val2 / val1        -- Ohmic resistance
BCoilLfemm = val3 / val1        -- Self-inductance

--////////////////////////////////////////////////////////////////////////
--// Calculate the time constants
--////////////////////////////////////////////////////////////////////////
BCoilRtotal = BCoilRfemm + 0.02
TC_RL = BCoilLfemm / BCoilRtotal
TC_RC = BCoilRtotal * Capacitance
TC_Ratio = TC_RL / TC_RC

-- Display the interim results in the Lua window
print("NumLayers=",NumLayers, "  AWG=",AWGGauge)
print("TC_Ratio=",TC_Ratio)
```

```
        -- Write the results to the file
        write(handle,"AWGGauge=,",AWGGauge,",")
        write(handle,"NumLayers=,",NumLayers,",")
        write(handle,"Rotal=,",BCoilRtotal,",")
        write(handle,"L(uH)=,",BCoilLfemm*1000000,",")
        write(handle,"TC_RL(s)=,",TC_RL,",")
        write(handle,"TC_RC(s)=,",TC_RC,",")
        write(handle,"TC_Ratio=,",TC_Ratio,"\n")

        mo_close()
        mi_close()
    end
end

-- Close out the Lua program
closefile(handle)
messagebox("All done.")
```