An OpenFoam analysis

The Joukowski airfoil at different viscosities

The transformations which generate a Joukowski-type airfoil were described in an earlier paper, entitled *The Joukowski airfoil in potential flow, without using complex numbers*. The general form of the Joukowski-type transformation, in which both translation distances are non-zero, was used. Expressions for the net force and pitching moment were developed. We found that: (i) the net aerodynamic force is entirely lift, which is to say that the direction of the force is at right angles to the relative airflow far upstream, and (ii) the magnitude of the lift is proportional to the sine of the angle of attack.

For use as a numerical example in the earlier paper, a Joukowski-type airfoil was considered whose upper surface closely resembles the NACA 2412 profile and whose chord is equal to the average chord on early versions of the Cessna 172.

In this paper, we will use the OpenFoam computational fluid dynamics program to calculate the forces on the same Joukowski airfoil we used as the numerical example before. For the sake of certainty, the generating parameters of this airfoil are:

$$\left.\begin{array}{l} R = 0.4051 \text{ meters} \\ f = 0.03069 \text{ meters} \\ g = 0.02032 \text{ meters} \\ b = 0.3672 \text{ meters} \end{array}\right\} \quad (1)$$

where the (p, q) co-ordinates of points on the surface of the airfoil have a 1:1 correspondence with points on the circle described by:

$$x_{\odot}^2 + y_{\odot}^2 = R^2$$

which is translated by:

$$x_s = R\cos\theta_{\odot} - f$$
$$y_s = R\sin\theta_{\odot} + g$$

and then inverted by the Joukowski transformation:

$$p = x_s + \frac{b^2 x_s}{x_s^2 + {y_s}^2}$$
$$q = y_s - \frac{b^2 y_s}{x_s^2 + {y_s}^2}$$

The most useful characteristic of the Joukowski transformation is that it is conformal, and so preserves angles locally. Scalar fields whose gradients are perpendicular at all points are transformed by the Joukowski transformation in a way which leaves them perpendicular at all points. This is very useful when the fluid flow is potential (incompressible, irrotational, steady and two-dimensional), since the defining characteristic of potential flows is the orthogonality of the velocity and "potential" fields.

The purpose of this paper is to compare the analytical solution from the earlier paper with numerical results found using OpenFoam. This is my first foray into OpenFoam and I would like to ensure that my use of the program is not too far wrong.

Meshing using GMesh

Computational fluid dynamics applies the equations for the fluid flow to discrete points in space (and in time as well if one is looking at a non-steady state, or transient, flow). The surface of the airfoil needs to be divided into short segments, the airfoil itself needs to be placed inside some bounded region of space, which I will call a virtual wind tunnel, and the space to be occupied by the fluid needs to be discretized into a mesh of small elements. I used the program GMesh to do these things. For a two-dimensional case like this one, GMesh produces a mesh of small triangular prisms which extend from the front face of the virtual wind tunnel through to the back face.

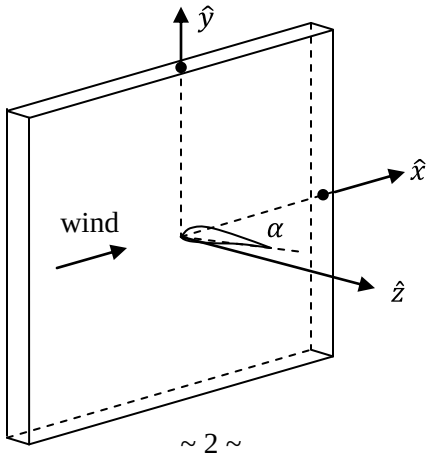
GMesh can take its instructions from a text file which contains the necessary commands in the prescribed syntax. GMesh will write a second text file with lists of the vertices, faces and tetrahedra (or prisms) in a format which OpenFoam can understand.

To assist in preparing the input text file, I wrote a program in Visual Basic 2010 Express which is listed in Appendix "A". The program starts with the parameters of the Joukowski airfoil in Equation (1) above. It generates the co-ordinates of the upper and lower surfaces. To be precise, it generates one million pairs of co-ordinates on the two surfaces, from among which it selects the pairs which lie closest to the spacing requirement specified by the user. The user can specify whether the points should be equally spaced or weighted towards the leading and trailing edges. [The one million points are generated at angles equally spaced around the source circle of the Joukowski airfoil. The corresponding points after the transformations will not be equally spaced along the surface of the airfoil.] The user can also specify the number of segments into which to divide the upper and lower surfaces. The default is 1000 segments, or panels, per surface and the runs whose results are shown below are based on this number of segments.

Appendix "B" is a listing of certain parts of the file produced by GMesh in the case where the airfoil is rotated to an angle of attack of 5°. Similar files were produced for angles of attack ranging from -5° to 25° at two-degree intervals. A mesh is produced for a given geometry, not for a given flight condition. By this, I mean that the same mesh is used when the angle of attack is 5°, whether the air speed is 100 mph or 120 mph, or whether the altitude is sea level or 10,000 feet. The wind condition is set by the parameters given to OpenFoam; the geometry is set by the parameters given to GMesh.

The virtual wind tunnel

The virtual wind tunnel is a rectangular parallelepiped. It is very thin – only one millimeter of the wing span is modeled. The origin of the co-ordinate frame of reference is set close to the geometric center of the parallelepiped. This is convenient because the co-ordinates of the points on the surface of the airfoil are generated in such a way that the leading edge has co-ordinates (0, 0). The axes of the co-ordinate frame of reference and the wind direction are related as shown in the following figure.



The relative wind is assumed to flow in the direction of the positive $\hat{x}$-axis. The $\hat{y}$-direction is "up" and the $\hat{z}$-axis completes a standard right-handed co-ordinate frame of reference. The thickness of the wind tunnel in the $\hat{z}$-direction has been exaggerated a little in the figure for clarity's sake. The origin is located halfway through the thickness, so the front and back faces of the wind tunnel are located at $z = \pm 0.0005\ m$, respectively. Since this is a two-dimensional analysis, it will not be necessary to solve the equations for the fluid flow in the $\hat{z}$-direction, so the front and back faces are defined as "empty" patches in OpenFoam's `/constant/polyMesh/boundary` file.

It is convenient to specify the length and height of the wind tunnel in terms of the length of the airfoil's chord. All of the cases described below were run with the following wind tunnel dimensions:

- distance from the leading edge / origin to the top face is 5 chord lengths,
- distance from the leading edge / origin to the bottom face is 5.5 chord lengths,
- distance from the leading edge / origin to the upwind face ("Inlet") is 5 chord lengths and
- distance from the leading edge / origin to the downwind face ("Outlet") is 6 chord lengths.

The `/constant/polyMesh/boundary` file for the 5° angle of attack case (which I will sometimes call the base case) is listed in Appendix "C". As stated already, the front and back faces of the wind tunnel are defined as "empty" patches. The top and bottom faces of the wind tunnel are defined as "symmetryPlanes", so that the pressure and velocity should be the same all along the face once the solution converges. This is a less demanding requirement than setting specific values on these faces, which gives the numerical routine more flexibility during each iteration while still converging to the same end. The pressure was set to zero across the Inlet and Outlet. Since the forces on the airfoil arise from differences in pressure at various points on the surface, the addition or subtraction of some constant value to the static pressure does not affect them. For convenience, the upwind and downwind static pressures were set to zero. The wind velocity was specified across the Inlet and Outlet as 44.704 meters per second, equal to 100 miles per hour, in the $\hat{x}$-direction.

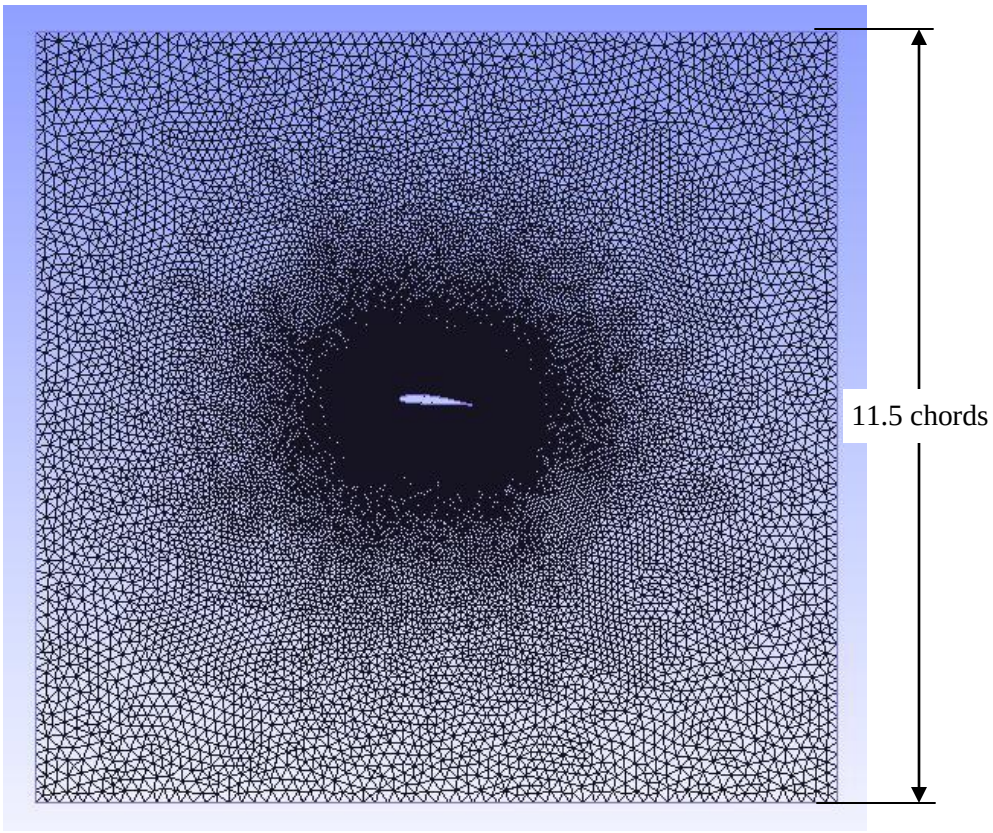
The characteristic lengths

GMesh permits the user to specify a "characteristic length" for every vertex, or point, that is defined inside or on the virtual wind tunnel. When GMesh discretizes the fluid volume into small elements, it tries to arrange things so that the mesh elements have edge-lengths equal to the characteristic length in and around the vertex, or point, at which they are defined. GMesh then interpolates the sizes of the elements elsewhere in the mesh to marry with the sizes at the vertices.

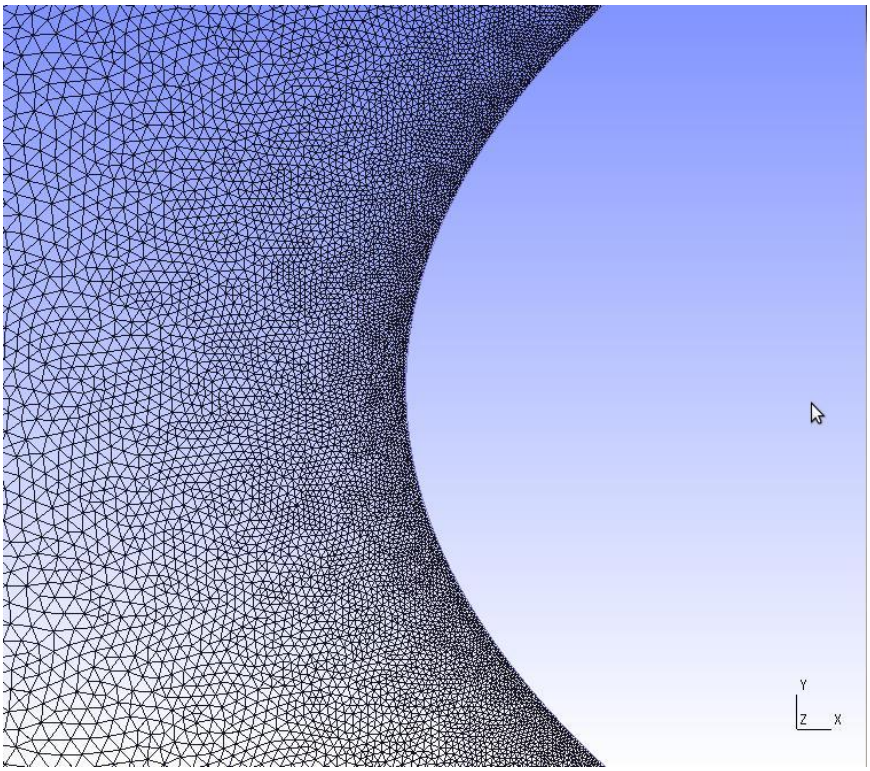
In all the runs done for this paper, the characteristic lengths were set as follows:

- the characteristic length on the surface of the airfoil is 0.2 millimeters and
- the characteristic length on the surface of the wind tunnel is 0.25 meters.

Because the wind tunnel is so thin, the tetrahedral mesh elements which GMesh generates by default in three dimensions are reduced to prisms, whose long axis extends straight through from the front face to the back face. The following figure shows the mesh at the largest scale, seen from the left side of the wind tunnel, when the airfoil is rotated to a 5° angle of attack. The figure after that shows the detail of the mesh in the vicinity of the leading edge. There are about 1,250,000 prisms in the mesh.



Mesh in wind tunnel



Mesh near leading edge

The files in the `/constant/` subdirectory

The `/constant/` subdirectory in the case directory holds values which will be constant throughout a single OpenFoam simulation. In the cases run for this paper, the subdirectories and files inside the `/constant/` subdirectory are as follows.

```
CaseDirectoryName/
|
|---constant/
|    |
|    |---polyMesh/
|    |    |
|    |    |---boundary
|    |    |---cellZones
|    |    |---faces
|    |    |---faceZones
|    |    |---neighbour
|    |    |---owner
|    |    |---points
|    |    |---pointZones
|    |
|    |---sets/
|    |    |
|    |    |---Internal
|    |    |---nonOrthoFaces
|    |
|    |---RASProperties
|    |---transportProerties
```

`OpenFoam creates the polyMesh/` and `sets/` subdirectories, by which I mean that they are not created by the user. OpenFoam creates them when it converts GMesh's mesh file into its own syntax, which it does in response to the command `gmshToFoam`. The `polyMesh/boundary` file, which I mentioned above, identifies the faces of all the mesh elements which comprise the physical boundary of the fluid in the wind tunnel. The other files in the `polyMesh/` subdirectory are lists, of the co-ordinates of the points, of the sets of points which constitute the faces of the mesh elements, of the mesh elements which have common faces, and so on. The file `sets/Internal` identifies the faces of all the mesh elements which are not located on the physical boundary of the fluid, but are instead located inside the fluid.

The file `sets/nonOrthoFaces` is an unfortunate file. Its presence indicates that there are deficiencies in the mesh. This particular file contains a list of the faces which OpenFoam judges to be severely non-orthogonal. After a GMesh mesh has been converted into OpenFoam's syntax, it is customary to run the command `checkMesh`. This will prompt OpenFoam to review the mesh and its constituent elements, looking for problems with the geometry which could adversely affect the accuracy or convergence of the numerical routines. If OpenFoam finds faces which are severely non-orthogonal, it will write a list of those faces into a file named `sets/nonOrthoFaces`, as happened here. OpenFoam looks for other deficiencies as well, such as highly-skewed elements, unclosed boundaries, and so on. For each type of deficiency, OpenFoam will write a list of the problematic entities into a file with an appropriately descriptive name. Of course, for a good quality mesh, there would be no deficiency files at all.

For some reason, the mesh which GMesh produces for the Joukowski airfoil has four faces which OpenFoam judges to be severely non-orthogonal. The meshes for all angles of attack had the same four

such faces. I was not able to correct this problem. When GMesh generates a mesh for a two-dimensional geometry, it does not allow any optimization. Fortunately, despite the existence of these four faces, GMesh reported "`Mesh OK`", and I proceeded with the imperfect mesh.

The files `constant/RASProperties` and `constant/transportProperties` are created by the user. The former specifies the mathematical model of the turbulence to be used and the latter specifies the transport properties of the fluid. In this study, I have chosen to use the Spalart-Allmaras turbulence model, which is said to be pretty good for two-dimensional flows. I have assumed that the fluid is Newtonian and, as a starting point, I have used the viscosity and density of air at sea level.

Appendix "D" is a listing of these two files for the base case.

The files in the `/system/` subdirectory

The `/system/` subdirectory in the case directory holds the settings which control the execution of an OpenFoam run. In the cases run for this paper, the subdirectories and files inside the `/system/` subdirectory are the following.

```
CaseDirectoryName/
|
|---system/
|   |
|   |---controlDict
|   |---decomposeParDict
|   |---fvSchemes
|   |---fvSolution
```

The file `decomposeParDict` controls the decomposition of the geometry into separate pieces, in preparation for execution by eight processors running in parallel. The file `fvSchemes` specifies the mathematical routines to be used to approximate the equations in their discretized form. The file `fvSolution` specifies how the physical variables should be derived from the equations of motion and how they should be permitted to change during the process of converging to a solution.

Appendix "E" is a listing of these four files for the base case.

It should be noted that the file `controlDict` includes the declaration of a function which prints out the pressure and viscous forces and moments after each iteration.

The files in the `/0/` subdirectory

The `/0/` subdirectory in the case directory holds the initial conditions for an OpenFoam simulation. Although the simpleFoam solver seeks a steady-state solution, for which there are no initial conditions *per se*, the "initial conditions" are treated as the starting estimates of the physical variables for use in the first iteration. The physical variables include the usual suspects – the pressure p, the velocity field U and the viscosity nu (or nut) – and also one more variable, nuTilda, which is a parameter used in the Spalart-Allmaras turbulence model.

In the cases run for this paper, the files inside the `/0/` subdirectory are as follows.

```
CaseDirectoryName/
|
|---0/
|   |
|   |---p
|   |---U
|   |---nut
|   |---nuTilda
```

I have attacked as Appendix "F" a listing of these four files for the base case.

The pressure file (`0/p`) and velocity file (`0/U`) are pretty straight-forward. The pressure is set to zero on the Inlet and Outlet faces of the wind tunnel. The wind velocity is set to 100 miles per hour on those faces, as well. The actual surface of the airfoil, which is a physical "wall" as far as OpenFoam is concerned, is treated as a no-slip wall on which the velocity is zero.

The nut and nuTilda files are a little different and contain the user's (that is, my) estimates for these two variables. There are a couple of rules of thumb which can be used to come up with these estimates. I took guidance from certain webpages of "CFD-Wiki, the free CFD reference" at www.cfd-online.com.

A parameter called the "turbulence intensity" I is defined for the Inlet of the virtual wind tunnel. It is intended to measure the turbulence in the flow of the fluid as it approaches the body of interest. The webpage gives some helpful examples. I quote:

1. "High-turbulence case: High-speed flow inside complex geometries like heat-exchangers and flow inside rotating machinery (turbines and compressors). Typically the turbulence intensity is between 5% and 20%.
2. "Medium-turbulence case: Flow in not-so-complex devices like large pipes, ventilation flows, etc., or low speed flows (low Reynolds number). Typically the turbulence intensity is between 1% and 5%.
3. "Low-turbulence case: Flow originating from a fluid that stands still, like external flow across cars, submarines and aircrafts. Very high-quality wind-tunnels can also reach really low turbulence levels. Typically the turbulence intensity is very low, well below 1%."

A parameter called the "turbulence length scale" l is defined as the estimate of the size of the largest-scale eddies in the turbulent flow. As a guide, the webpage says,

> "It is common to set the turbulence length scale to a certain percentage of a typical dimension of the problem. For example, at the inlet to a turbine stage a typical turbulence length scale could be say 5% of the channel height. In grid-generated turbulence the turbulence length scale is often set to something close to the size of the grid bars."

In this study, the airfoil will be simulated at a range of angles of attack, including high angles of attack at which there will be massive separation of the airfoil from the surface. The eddies which arise in these cases will likely be an appreciable fraction of the length of the chord, perhaps 10% or more.

In a page entitled *Turbulence free-stream boundary conditions*, the turbulence intensity and turbulence length scale are combined into the following estimate for nuTilda ($\tilde{\nu}$):

$$\tilde{\nu} = \sqrt{1.5} U_{inf} I l \quad (2)$$

This webpage makes a remark about the Spalart-Allmaras turbulence model, which is therefore relevant for this study, that:

> "Ideally with the Spalart-Allmaras model $\tilde{\nu} = 0$, but some solvers can have problem with that so $\tilde{\nu} \leq \tilde{\nu}/2$ can be used. This is if the trip term is used to "start up" the model. A convenient option is to set $\tilde{\nu} = 5\nu$ in the freestream. The model then provides fully turbulent results and any regions like boundary layers that contain shear become fully turbulent."

In the base case, and all the other runs made using the viscosity at sea level, I used the following estimates:

$$\left.\begin{array}{c} U_{inf} = 44.7\ m/s \\ I = 5\% \\ l = 0.1\ meter \\ \rightarrow \quad \tilde{\nu} = 0.274 \end{array}\right\} \quad (3)$$

Then, in the `0/nuTilda` file, I defined the freestream $\tilde{\nu}$ as five times this value. In the `0/nut` file, I defined the viscosity as one-tenth of this value.

These are initial settings for the simulation. As the simulation proceeds, OpenFoam uses in own interim calculations to improve the estimates. It is helpful if the initial estimates are as accurate as possible, but it is not fatal to the procedure if they are wrong.

The results from the OpenFoam simulation of the base case

The base case assumes the kinematic viscosity of air at sea level (1.4604×10^{-5} m^2/s), the density of air at sea level (1.225 kg/m^3) and a relative wind of 100 miles per hour (44.704 m/s). In this section, I will describe the results from the OpenFoam simulation and compare them with the results found analytically in the earlier paper, in which potential flow was assumed.

The base case required 18,394 iterations to converge to the point where the residual errors for the physical variables were all less than 10^{-5}. At convergence, OpenFoam reported that the pressure and viscous forces and mechanical moments were as follows:

$$\left.\begin{array}{lll} \text{pressure force downwind} & F_p\hat{x} & = 0.0651\ \text{N} \\ \text{pressure force upwards} & F_p\hat{y} & = 1.3303\ \text{N} \\ \text{viscous force downwind} & F_v\hat{x} & = 0.0154\ \text{N} \\ \text{viscous force upwards} & F_v\hat{y} & = -0.0004\ \text{N} \\ \text{pressure moment} & M_p\hat{z} & = 0.6644\ \text{Nm} \\ \text{viscous moment} & M_v\hat{z} & = -0.0006\ \text{Nm} \end{array}\right\} \quad (4)$$

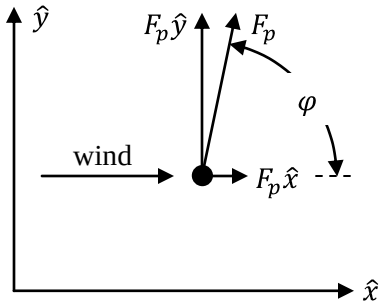
The forces acting across the wind tunnel, in the $\hat{z}$-direction, were vanishingly small, bordering on machine precision. As a result, the only non-zero mechanical moments were those in the $\hat{z}$-direction, which tend to torque the leading edge down when they are algebraically-positive or to torque the leading edge up when they are algebraically-negative.

Let's first compare the magnitudes of the pressure and viscous forces. Since an airfoil is usually designed primarily to develop "lift", we expect and see here that the pressure force acting upwards is much greater than the pressure force acting downwind. Like most streamlined bodies, the viscous forces are small compared with the pressure forces. At a 5° angle of attack, the boundary layers which develop on the

upper and lower surfaces resist the flow of air much more in the direction parallel to the surface than in the direction perpendicular to it. As a result, the magnitude of the viscous force acting downstream is much larger than the magnitude of the viscous force acting upwards or downwards. And, in fact, what little viscous force there is in the direction perpendicular to the relative wind, is directed downwards. This makes sense intuitively – the airfoil is rotated downwards at its aft end, so the air which drags itself along the surface is actually pulling downwards, hence the algebraically-negative viscous force $F_v\hat{y}$.

Since the pressure forces are greater than the viscous forces, the mechanical moment they cause is greater than the mechanical moment the viscous forces cause. The algebraically-positive net moment is in the direction which forces the nose of the airfoil down.

Let's now set aside the viscous forces and look only at the pressure forces. We can combine the two components of the pressure force into its total magnitude and its direction, which we will measure as its angle φ with respect to the $\hat{x}$-axis. The following figure shows the geometry.



In the figure, the airfoil is shown as simply a dot. The components of the pressure force are combined using the Pythagorean Theorem, thus:

$$\begin{aligned} F_p &= \sqrt{\left(F_p\hat{x}\right)^2 + \left(F_p\hat{y}\right)^2} \\ &= \sqrt{0.0651^2 + 1.3303^2} \\ &= 1.3319 \text{ N} \end{aligned} \qquad (5)$$

and the angle φ can be found using the inverse tangent function:

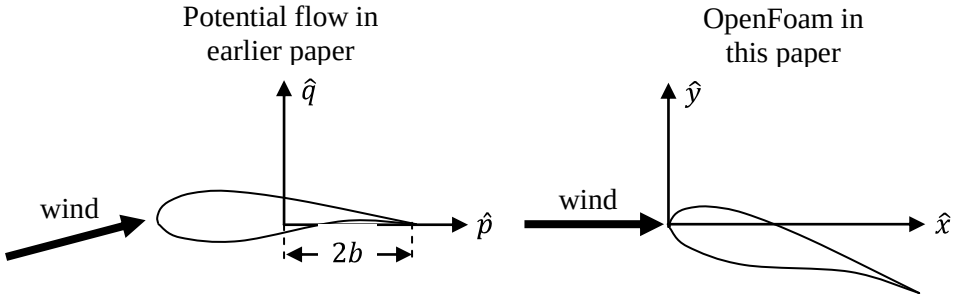
$$\begin{aligned} \varphi &= \tan^{-1}\left(\frac{F_p\hat{y}}{F_p\hat{x}}\right) \\ &= \tan^{-1}\left(\frac{1.3303}{0.0651}\right) \\ &= 87.198° \end{aligned} \qquad (6)$$

The total pressure force in Equation (5) is the total pressure force acting on the section of the airfoil inside the virtual wind tunnel. Recall that the wind tunnel is only one millimeter thick. A section of airfoil one unit length, or one meter, in span, would experience 1000 times as much force, or 1,331.9 N. The angle in Equation (6) is about 2.8° past the vertical to the relative wind, where "past" means biased downwind.

We can compare these figures to the ones we found analytically. When this airfoil was analyzed in a potential flow, the pressure force was found to be exactly perpendicular to the relative wind. The total

pressure force was found to be 1,689.3 N. Of course, potential flow assumes, among other things, that the viscosity is zero. The OpenFoam simulation was run assuming the airfoil was cruising in real air at sea level. Perhaps one should be pleasantly surprised that the simplified assumptions of potential flow give a total pressure force within 29% of that generated using OpenFoam.

Let's turn now to the mechanical moment, which in the case of airfoils is called the pitching moment. The definition of a moment must include the specification of a rotation axis. It happens that the earlier potential-flow analysis and the OpenFoam simulation done in this paper use different rotation axes. Before we can compare the moments, we need to reconcile the difference between the rotation axes. They are compared in simplified form in the following figure.

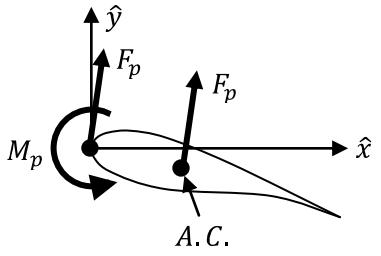


In the OpenFoam analysis (in this paper), the wind approaches the airfoil along the $\hat{x}$-axis and encounters the mesh, which was constructed around the airfoil after its trailing edge had been rotated downwards by the 5° angle of attack. The airfoil was positioned with its leading edge at the origin of the co-ordinate frame and the airfoil was rotated around the origin into the desired angle of attack. The moment is calculated around the origin / leading edge. This configuration is shown in the diagram on the right-hand side of the figure.

In the potential-flow analysis (in the earlier paper), the airfoil was generated by a Joukowski transformation of points on the circumference of a circle into the $\hat{p}$-$\hat{q}$ plane. The transformation is such that the trailing edge of the airfoil lies a distance $2b$ to the right, along the $\hat{p}$-axis, of the origin of the $\hat{p}$-$\hat{q}$ plane. The relative wind approaches the airfoil from the lower left, at an angle below the negative $\hat{p}$-axis equal to the desired angle of attack. The moment is calculated around the origin, which is approximately halfway along the chord. This configuration is shown in the diagram on the left-hand side of the figure.

The physics of the two configurations is the same. We just have to ensure that we take into account the difference between the origins around which the moments were calculated.

In the earlier paper, we looked a more useful measure of the pitching moment than the value of $M_p\hat{z}$ itself. We looked for the physical point along the chord, somewhere between the leading edge and the trailing edge, at which we could apply the total pressure force without any compensating moment. This is the "aerodynamic center". We can do the same thing with the results from the OpenFoam runs. Consider the following figure.



When the total pressure force is applied at the leading edge, which is the origin in the OpenFoam frame of reference, the moment $M_p\hat{z}$ is equal to 0.6644 Nm, for the one millimeter section in the wind tunnel, or 664.4 Nm per meter of span. We will translate the point of application of the force towards the trailing edge until we find the point at which the accompanying moment is zero.

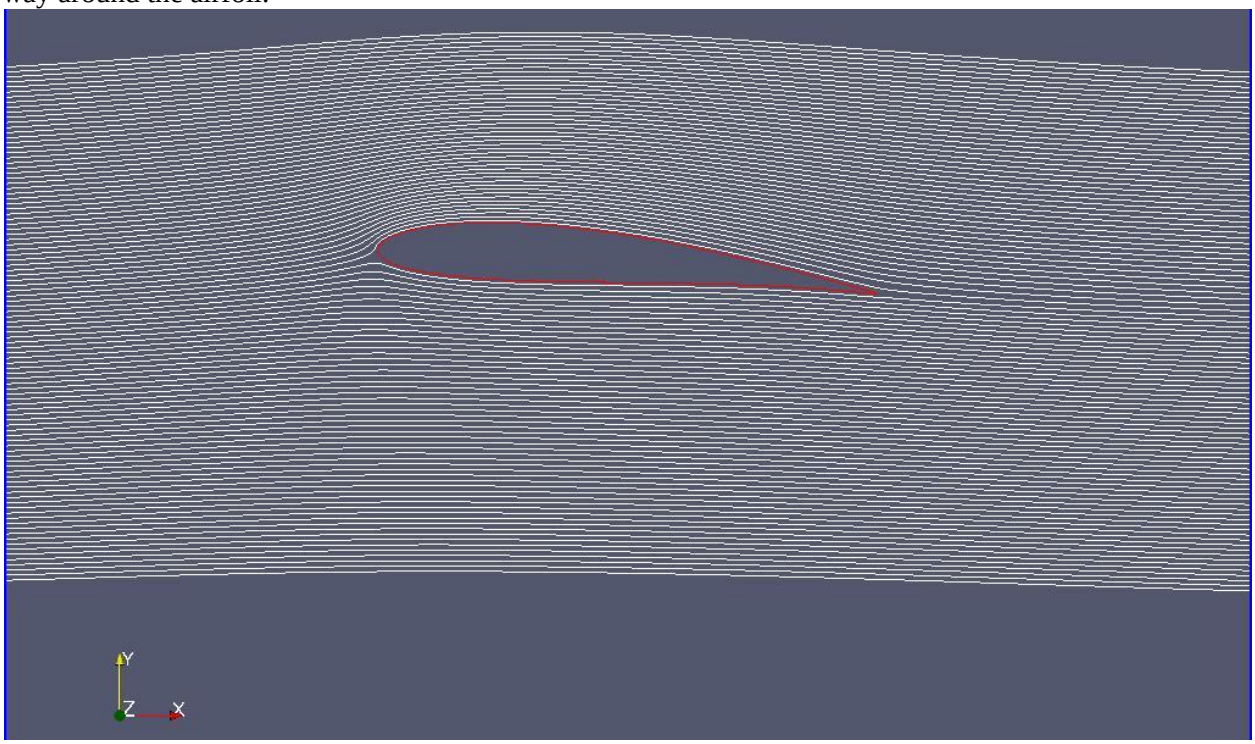
Moving some distance D_s along the chord is equivalent to moving a distance $D_x = D_s \cos\alpha$ along the $\hat{x}$-axis and a distance $D_y = -D_s \sin\alpha$ along the $\hat{y}$-axis, where α is the angle of attack. Distance Dx will be the moment arm through which the vertical component of the pressure force $F_p\hat{y}$ exerts a nose-down moment around the leading edge. Similarly, distance $|D_y|$ (the positive distance) will be the moment arm through which the horizontal component of the pressure force $F_p\hat{x}$ also exerts a nose-down moment around the leading edge. When translation by distance D_s gives a combined nose-down moment equal to zero, we will have arrived at the aerodynamic center. This will happen mathematically when:

$$\begin{aligned} M_p &= D_x F_p\hat{y} + |D_y| F_p\hat{x} \\ &= D_s\left(F_p\hat{y}\cos\alpha + F_p\hat{x}\sin\alpha\right) \\ \rightarrow \quad D_s &= \frac{M_p}{F_p\hat{y}\cos\alpha + F_p\hat{x}\sin\alpha} \end{aligned} \qquad (7)$$

The numerical values in the base case are:

$$\begin{aligned} D_s &= \frac{664.4\text{ Nm}}{(1330.3\text{N}\times\cos 5°) + (65.1\text{N}\times\sin 5°)} = 0.4992\text{ meters} \\ \rightarrow \quad \frac{D_s}{\text{chord}} &= \frac{0.4992\text{ meters}}{1.480\text{ meters}} = 33.7\% \end{aligned} \qquad (8)$$

It is traditional to express the distance from the leading edge to the aerodynamic center as a percentage of the length of the chord, in this case, as 33.7%. The following screenshot shows stream tracers flowing around the airfoil, based on the pattern of airflow calculated by OpenFoam. Stream tracers are, to all intents and purposes, the same as streamlines, but are not intended to have any connotation of zero viscosity. They are small "tubes", if you will, through which small bits of the fluid pass as the make their way around the airfoil.



The pattern of the airflow looks very similar to the pattern of the airflow we found when we examined this airfoil in potential flow. The source line for the stream tracers in the figure is a vertical line located two meters upwind from the leading edge. It extends from one meter below the leading edge to one-half meter above. The source line is located a very small distance, 0.1 millimeters, to the left side (towards us, on this side of the paper) of the central plane through the wind tunnel, simply to avoid any small perturbations which may be associated with the mesh. There are 200 individual stream tracers, generated at equally-spaced intervals along the source line.

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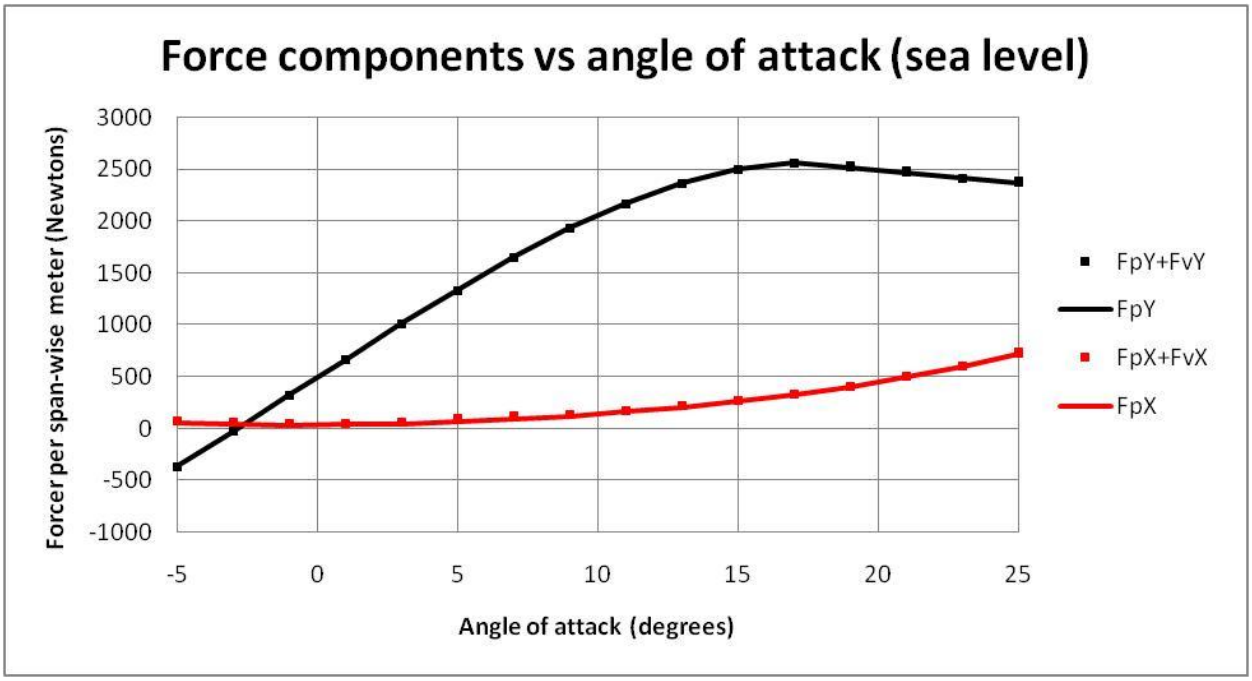
The results from the OpenFoam simulation using sea level air conditions

In this section, we will look at the results when the airfoil is positioned at various angles of attack, not just 5°. In all of these cases, the relative wind speed is 100 miles per hour and the density and viscosity of the air are their sea level values, per the U.S. Standard Atmosphere.

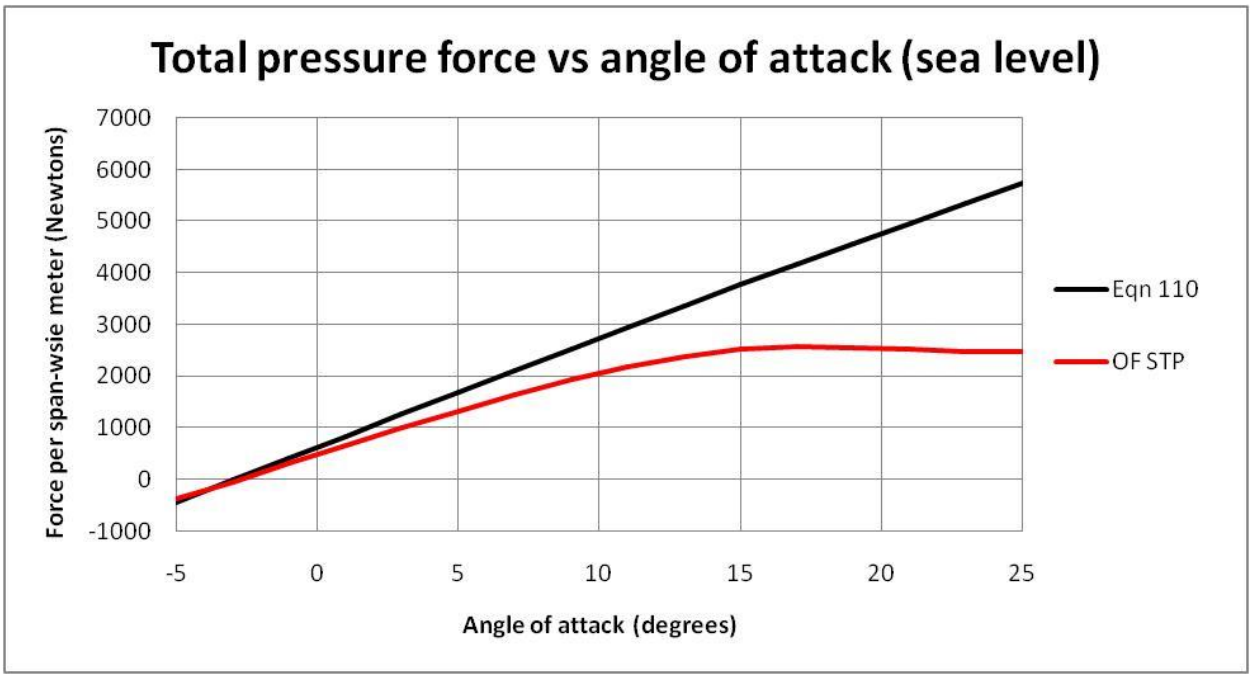
The following graph shows the components of the aerodynamic forces acting on the airfoil at a range of angles of attack from -5° to +25°. Four force components are shown. The black line and black dots are the forces acting in the vertical "up", or $\hat{y}$, direction. The red line and red dots are forces acting in the downwind, or $\hat{x}$, direction. The solid lines are the pressure forces only. Adding the viscous forces to the pressure forces gives the dots. All of the forces have been multiplied by a factor of 1000 to convert the one millimeter section of airfoil in the wind tunnel into a section one meter in span.



In the "up" direction, perpendicular to the oncoming wind, the viscous force is so small that one cannot really distinguish in the graph between the line (pressure force only) and the dots (pressure force plus viscous force). One can make the distinction in the downwind direction, where the viscous force adds a little to the pressure force, but the difference is quite small.

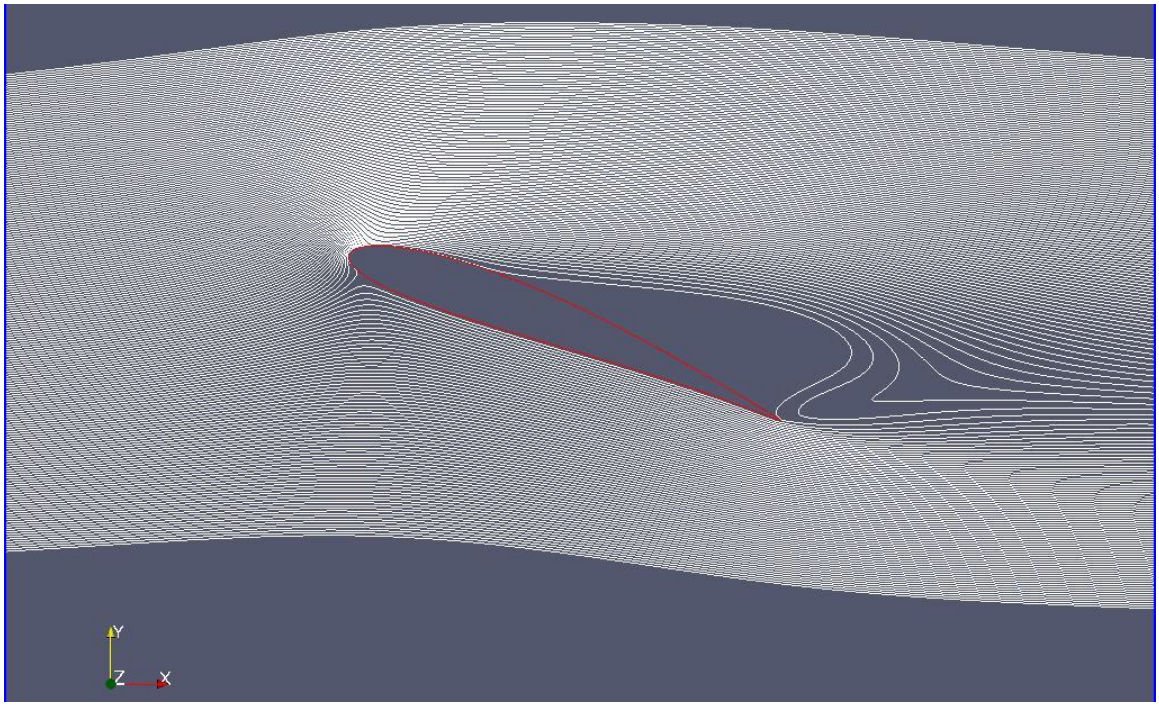
In the remainder of this section, we are going to ignore the effects of the viscous force. They are small but, just as importantly, the potential flow against which the OpenFoam results are being compared has no viscous effects at all.

The following graph shows the total pressure force, being the square root of the sum of the squares of the two components, versus the angle of attack. That is shown by the red line. The black line shows the comparable curve for potential flow, which was derived in the earlier paper.



A word about the legend: "OF" stands for OpenFoam and "STP" is the acronym for Standard Temperature and Pressure. The red line is the total pressure force calculated by OpenFoam at these angles of attack. The black line is the total pressure force calculated using Equation (110) in the earlier paper, in which the airflow was modeled as potential flow.

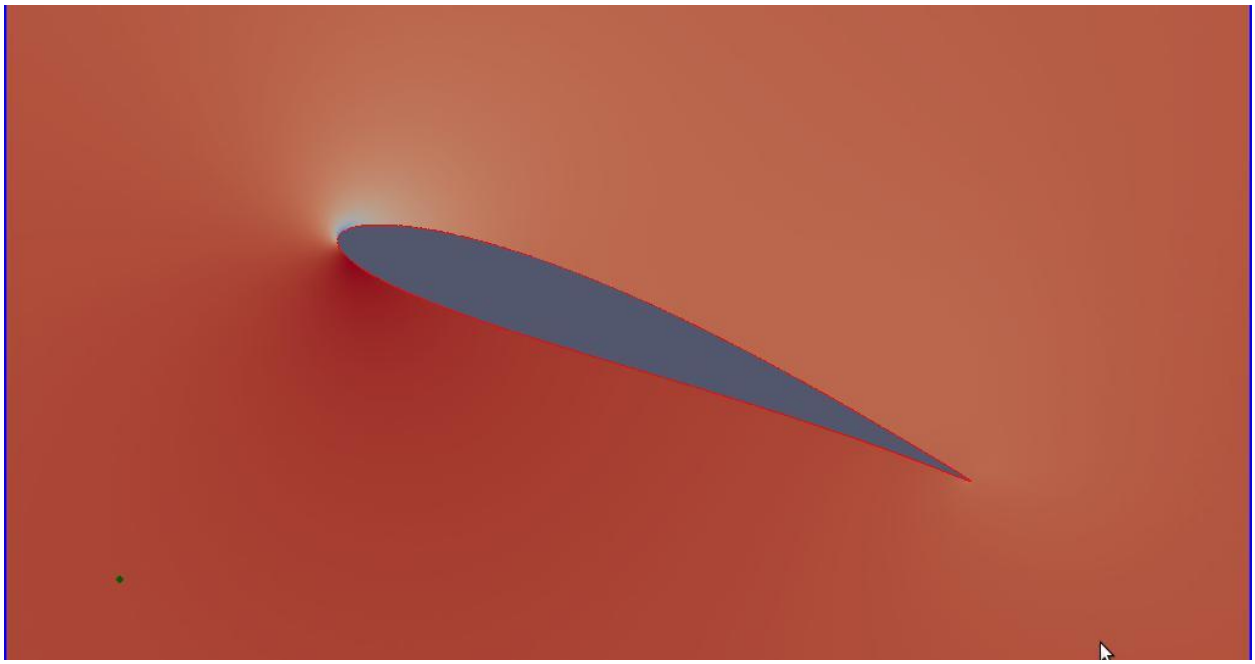
Up to about ten degrees of attack, the two curves are similar. At greater angles of attack, they are not. At the higher angles of attack, the air stops flowing smoothly over the full length of the upper surface of the airfoil and, instead, separates from the surface. This tendency is illustrated in the following screenshot, which shows the same set of stream tracers from above, but in the case when the angle of attack is 21°.



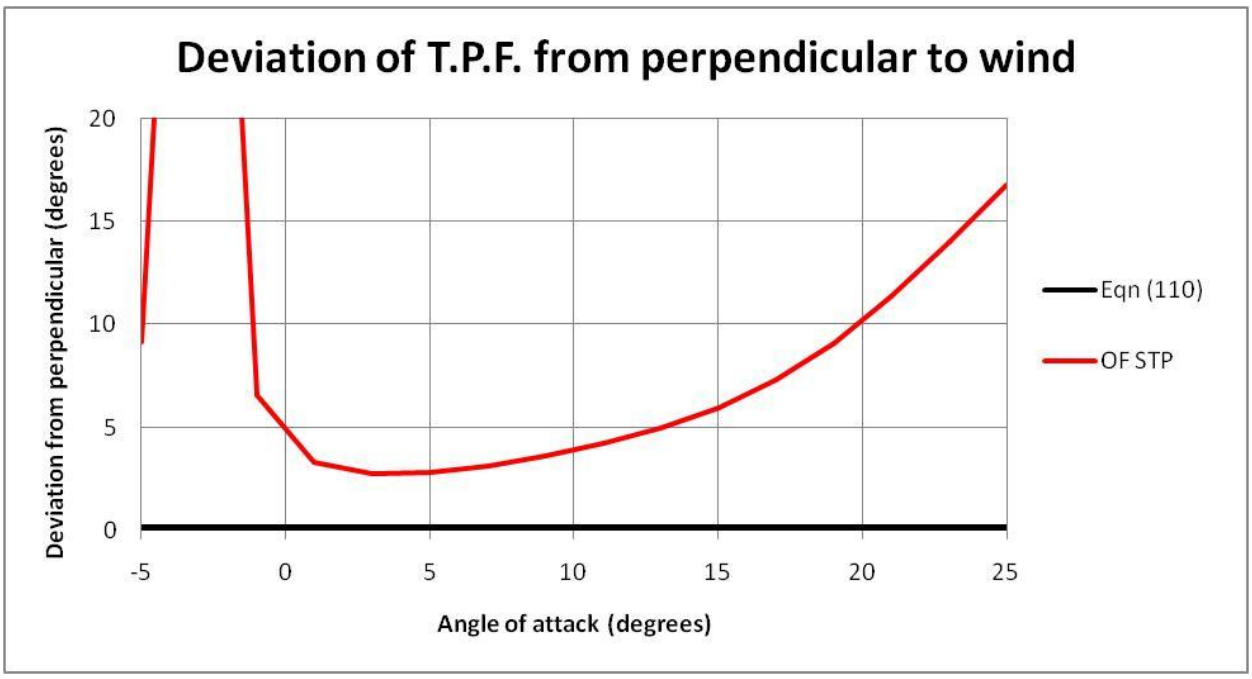
The airflow separates from the surface not far from the point of maximum thickness. It circles back towards the trailing edge, defining something like a bubble on the upper surface. There is air inside the bubble, and it will be moving violently around, in eddies of various scales. On the whole however, the air in the bubble will not be moving as fast as the air in the stream tracer which just bounds the bubble. This is important. With the right tweaking, or interpretation, Bernoulli's Principle still applies: the reduction in static pressure on the surface of the airfoil is proportional to the square of the air speed near the surface. When a high speed streamline or stream tracer runs along the upper surface, the static pressure is reduced, giving greater scope for the pressure acting on the lower surface of the airfoil to push upwards. If that high speed streamline or stream tracer is replaced with the more confused flow inside a bubble, the static pressure is higher than it otherwise would be, and the net lift on the airfoil is less than it otherwise would be.

When the angle of attack is modest, say less than ten degrees, a small increase in the angle of attack leads to a pattern of airflow where the speed of the air flowing over the top surface increases, leading to an increase in the lift generated by the airfoil. Once the airfoil has stalled, and the flow separated from the surface, further increases in the angle of attack just create more turbulence in the bubble, but not greater lift.

The following screenshot shots the static pressure on the surface of the airfoil at the 21° angle of attack. The pressure is relatively constant along the upper surface, except near the leading edge, where the air still flows at high speed close to the surface.



Let's look at the direction of the aerodynamic force. When we looked at the Joukowski airfoil in potential airflow, we found that the net aerodynamic force was always lift, and entirely lift, where lift refers to the component of the force perpendicular to the direction of the relative wind. Potential flow did not give rise to any component of pressure-induced drag. The components of the pressure force calculated by OpenFoam do include a small amount of drag. The relative amount of this drag can be inferred from the following graph, which shows the angle of the total pressure force with respect to the perpendicular to the relative wind.



Over the normal operating range of angles of attack, say, from 0° to 10°, the total pressure force (T.P.F.) is biased aft by between three and five degrees. Once the wing stalls, and the airflow separates from the surface, the induced drag continues to rise, at an increasing rate. [The figures for angles of attack less than zero need to be put into context. It is at angles of attack in this range that the $\hat{x}$- and $\hat{y}$-components of the pressure force change their algebraic sign – they "go through zero". When either component goes through zero, the angle it describes as a vector loses much of its significance. The fact that the two components go through zero at slightly different angles of attack broadens the range over which the "direction" of the force is not really a meaningful quantity.]

We can do as we did in the particular case of the 5° angle of attack, and express the pitching moment at these various angles of attack in terms of the location of the aerodynamic center along the chord. I am going to defer that exercise until after the following section.

Reducing the viscosity in the OpenFoam cases

A potential airflow, which we assumed when we analyzed the Joukowski airfoil in the earlier paper, has four physical characteristics. Three of these characteristics – that the flow is steady with respect to time, that it is two-dimensional in geometry and that the fluid is incompressible – are shared by the formulation of the OpenFoam cases. To begin with, the mesh was created with only two-dimensions. The simpleFoam solver itself includes the equations for steady, incompressible flow.

The remaining characteristic relates to viscosity. In a potential flow, the viscosity of the fluid is zero. In the OpenFoam cases described above, the viscosity was not zero; it was the viscosity of air at sea level in the U.S. Standard Atmosphere. It is possible to re-run the OpenFoam cases using other viscosities.

I hypothesized that the difference between the forces we calculated in the earlier paper and the forces calculated by OpenFoam are the result solely of the difference in viscosity. To test this hypothesis, I re-ran the base case twice, once with a viscosity of one-hundredth the sea level value and again with a viscosity of one-thousandth the sea level value. In preparation for these runs, the viscosity was changed

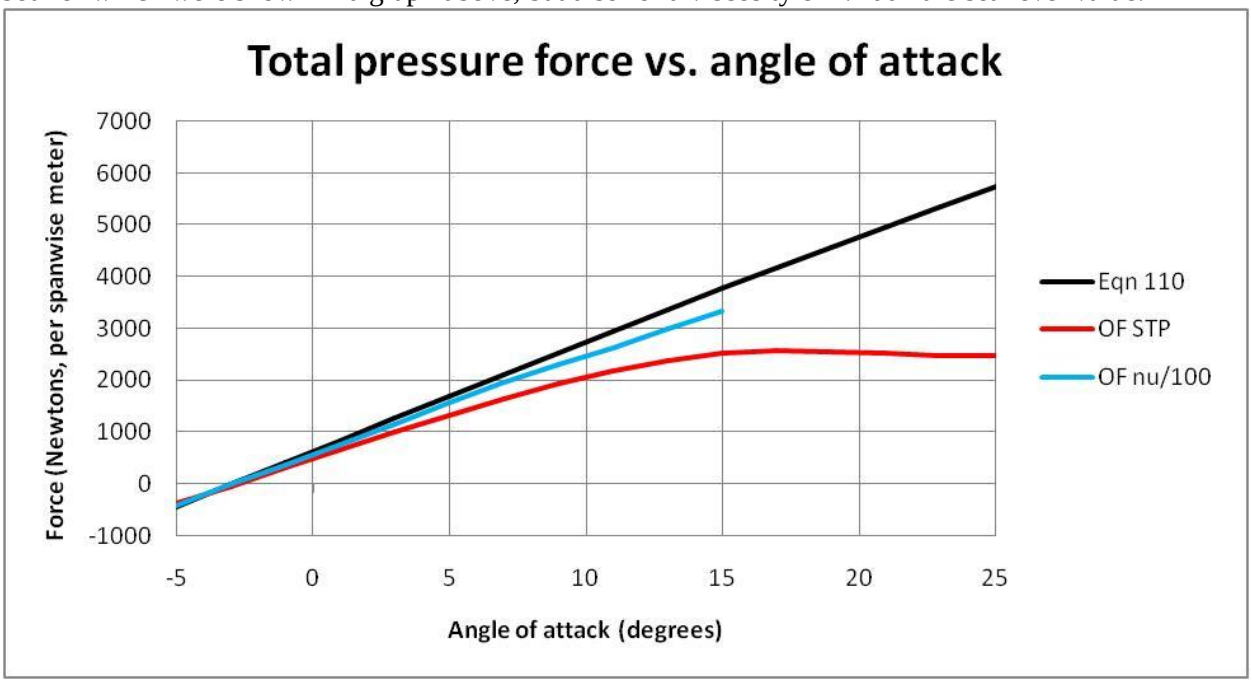
in three of the files in the case directory: in `constant/transportProperties`, in `0/nut` and in `0/nuTilda`.

The measure of viscosity defined in each file was divided by 100 for the first new run and by 1000 for the second new run. The results do, indeed, approach the results for the potential-flow case. The following table sets out the total pressure force and the angle of the total pressure force aft of the perpendicular to the relative wind, in four situations.

	OpenFoam at sea level	OpenFoam with 1/100th viscosity	OpenFoam with 1/1000th viscosity	Potential Airflow
Kinematic viscosity (m^2/s)	1.4604×10^{-5}	1.4604×10^{-7}	1.4604×10^{-8}	0
Total pressure force (N)	1,331.858	1,563.361	1,595.748	1,689.274
and as %	78.84%	92.55%	94.46%	100.00%
Deviation from vertical (°)	2.803°	0.255°	0.098°	0°

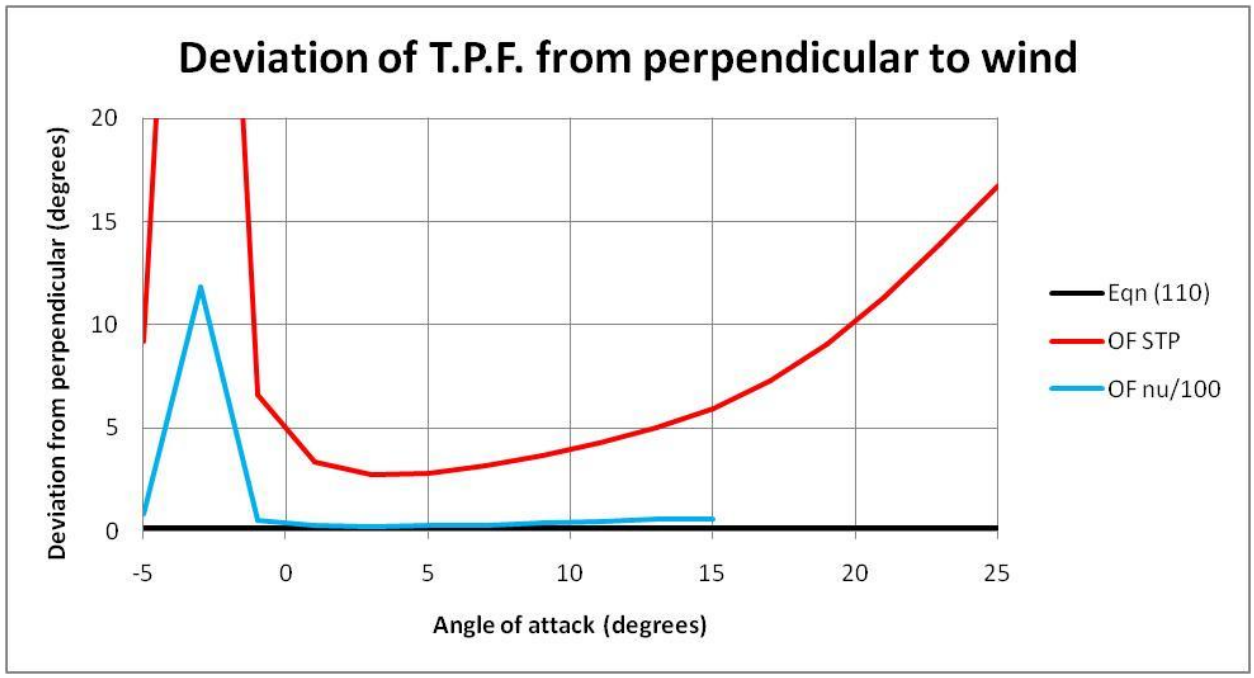
Both the total pressure force and the deviation angle approach the potential-airflow values as the viscosity is reduced. I would have liked to make runs with even lower viscosities, but was unable to do so. The numerical procedure seems to become more and more unstable as the viscosity is reduced. Take for example the case where the viscosity is 1/1000th of the sea level value. I had to reduce the under-relaxation parameters in the `/system/fvSolution` file to unusually low values to prevent the numerical routine from "blowing up". I set the under-relaxation parameter for the pressure to 0.2 and the under-relaxation parameter for the velocity to 0.4. The under-relaxation parameter is proportional to the fraction of the error in the physical variables by which they are adjusted prior to the next iteration. A low under-relaxation parameter causes only a small part of the error in each iteration to be "fed back" into the solution. This causes the solution to change in smaller steps, reducing the chances of a "blow-up", but at the expense of longer execution times. The run with 1/1000th of the sea level viscosity took 44,146 iterations to converge and was the only run at this viscosity which I was able to complete.

On the other hand, I was able to run a range of angles of attack at a viscosity of 1/100th of the sea level value. The following graph shows the total pressure force for potential airflow and sea level viscosity, both of which were shown in a graph above, but also for a viscosity of 1/100th the sea level value.



As in the previous graph, the black line is the total pressure force for potential airflow and the red line is the total pressure force from the first set of OpenFoam cases, run with sea level viscosity. The blue line is the total pressure force with 1/100th of sea level viscosity. It is much closer to the potential airflow result, and does not show any effects of stall / separation of flow, at least up to an angle of attack of 15°.

The following graph compares the deviation angle for the same three situations.

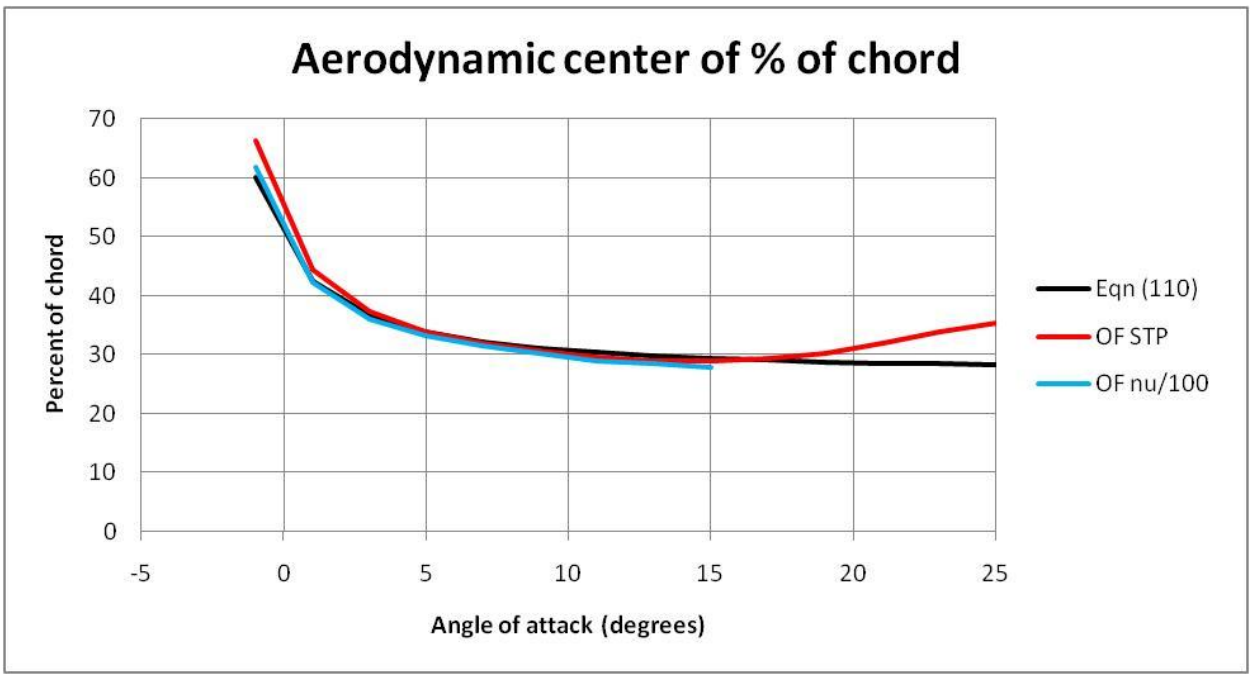


The black and red lines are the same as shown in a previous graph, and are the deviation angles for potential airflow and sea level viscosity, respectively. The blue line is the deviation angle at the reduced viscosity. It is much, much closer to the ideal potential airflow case. As before, the results for the angles of attack a few degrees below zero are unreliable.

The location of the aerodynamic center

This section will deal with the location of the aerodynamic center. To reiterate, the aerodynamic center is the location along the chord where the aerodynamic forces could be applied without the need for any compensating mechanical moment. We used Equation (7) above to calculate the location of the aerodynamic center for the base case. The same calculation was applied to the other cases run with sea level viscosity and to all the cases run with one-percent of sea level viscosity. The following graph shows the location of the aerodynamic center, expressed as a percentage of the length of the chord.

It is often said that the aerodynamic center of a cambered airfoil is near the quarter chord. The graph shows that there is a real basis for that statement.



The location of the aerodynamic center seems to depend less on the viscosity than the aerodynamic forces themselves do. The comparison is quite good, especially over the range of normal operating angles of attack.

y+

y+ is a dimensionless number. It is the ratio of the size of a mesh element which lies on the surface of the airfoil to the thickness of the boundary layer at the location of the mesh element. If an element in the mesh has a very large y+, say, more than 100, then the boundary layer extends only a small distance into the element, approximately 1% of its height. It is obvious that, in such a case, the average conditions of the air in the mesh element are not related at all to the conditions inside the boundary layer.

On the other hand, when y+ is small, say less than one, the mesh element is smaller in size than the boundary layer. The average conditions of the air inside the small mesh element are much more representative of the conditions inside the boundary layer than those inside the large mesh element.

The relative size of the mesh elements and the boundary layer is an important parameter when determining if the turbulence model being used, Spalart-Allmaras in this paper, is an appropriate physical model for the effects of viscosity. Every turbulence model includes some experimental data, and the model can be used safely only if the configuration being modeled is similar to the configuration which was used to collect the empirical data.

Turbulence models generally deal with variations in y+ using a two-pronged strategy. If the size of the mesh elements is small enough, say, y+ less than one, then the turbulence model will try to calculate the conditions in the boundary layer, or even in sub-layers in the boundary layer, directly. If the mesh is coarse, say, y+ greater than 30, then the model will use a "wall function" instead. The former approach is potentially more accurate, since an attempt is made to actually calculate the conditions in the boundary layer. The latter approach is limited to estimating effects at a larger scale. Most modern implementations of turbulence models for CFD allow the model to switch between the two strategies depending on the

local value of y+. OpenFoam's implementation of the Spalart-Allmaras turbulence model includes such an automatic switch.

In the base case defined in this paper, where the airfoil is held at a 5° angle of attack to a 100 mile per hour relative wind, the average y+ on the surface of the airfoil is 7.85. A separate value of y+ can be calculated for each mesh element which has a face on the surface of the airfoil. The values of y+ for the individual mesh elements ranged from a low y+ of 0.04 to a high y+ of 19.3. The Spalart-Allmaras model should prove satisfactory when used in this range.

In the cases run with $1/100^{th}$ of sea level viscosity, one expects that the reduced viscosity would cause the boundary layer to be much thinner and, given that the same mesh is used, the calculated values of y+ to be much higher. When the airfoil is held at a 5° angle of attack to a 100 mile per hour relative wind, and the viscosity is $1/100^{th}$ of its sea level value, the average y+ on the surface of the airfoil is 422, with a minimum of 3.5 and a maximum of 1183. This configuration is at the limits of the applicability of the Spalart-Allmaras turbulence model, which is said to decrease in accuracy once y+ exceeds 300. If time permitted, it would be wise to re-run one or more of these low viscosity cases with a finer mesh. More study is needed.

When the viscosity is reduced by another factor of ten, to $1/1000^{th}$ of its sea level value, the calculated y+ increases even further. The average y+ is found to be 3550, with a minimum of 18.4 and a maximum of 10,007. While y+ at some of the mesh elements on the surface seems satisfactory, the average mesh element is so much greater than 300 (the nominal maximum value of y+ which Spalart-Allmaras is intended to accommodate), that we cannot rely on the results of this run.

This concludes my OpenFoam analysis of the Joukowski airfoil.

Jim Hawley
June 2013

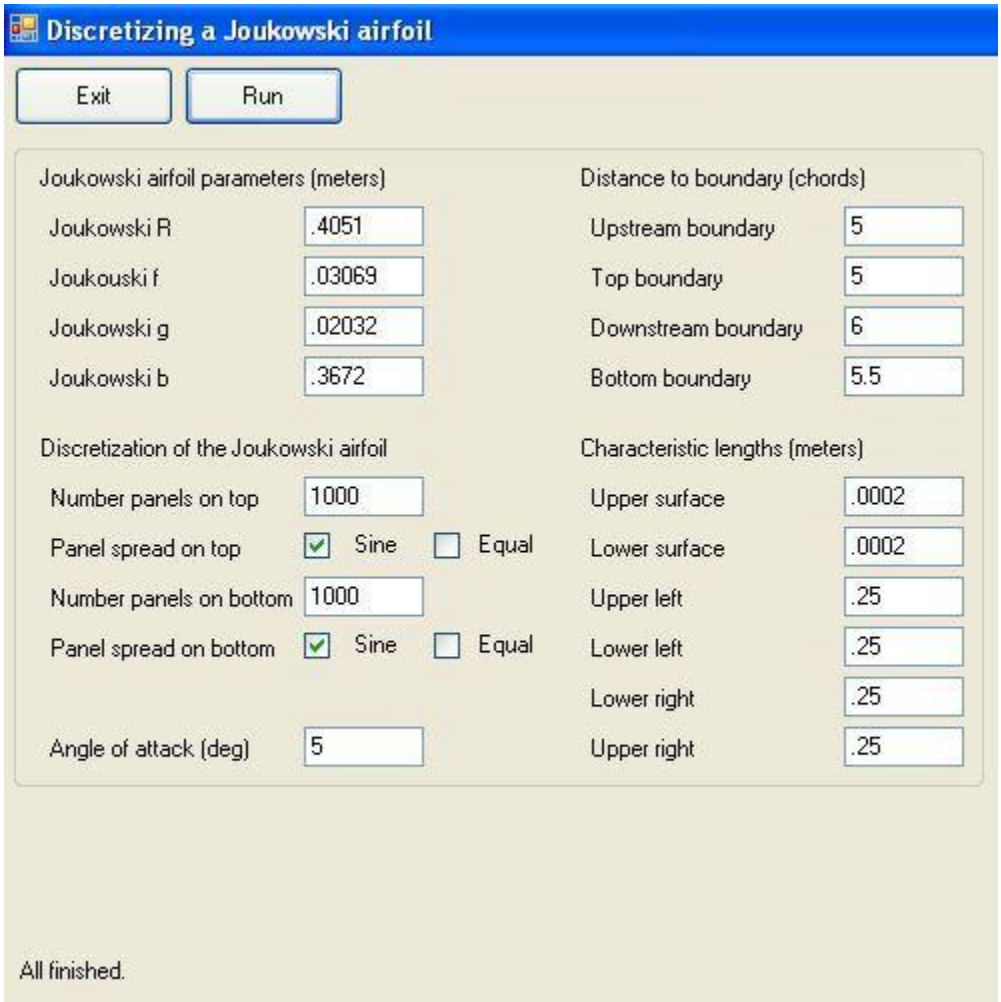
An email setting out errors and omissions would be appreciated.

Appendix "A"

A Visual Basic program to produce an input file for GMesh

The program consists of the main windows form (Class Main) and three modules. Class Main manages the GUI through which the user can change default values. Module Variables declares and sets values for certain global variables. Module Joukowski calculates the co-ordinates of points on the surface of the airfoil and module GMesh writes the necessary instructions to the output text file.

The following screenshot shows the input fields available in the GUI. The four parameters of the Joukowski airfoil are entered in the upper left quadrant. The number and spacing of points on the upper and lower surfaces are set in the lower left quadrant. The size of the surrounding virtual wind tunnel is set in the upper right quadrant. Note that the dimensions of the wind tunnel are set in terms of chord-lengths. The desired lengths of the edges of the tetrahedra (prisms) in the mesh are set in the lower right quadrant. These distances are specified in absolute units, being meters.



Listing of class Main

```
Option Strict On
Option Explicit On

' Discretizing a Joukowski airfoil

Public Class Main
    Inherits System.Windows.Forms.Form

    Public Sub New()
        InitializeComponent()
        With Me
            Name = ""
            Text = "Discretizing a Joukowski airfoil"
            FormBorderStyle = Windows.Forms.FormBorderStyle.FixedSingle
            Size = New Drawing.Size(1024, 720)
            CenterToScreen()
            Visible = True
            Controls.Add(buttonExit)
            Controls.Add(buttonRun)
            Controls.Add(GroupboxEdit)
            Controls.Add(DisplayText)
            PerformLayout()
        End With
    End Sub

    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////
    '// Principal controls and handlers.
    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////

    Public WithEvents buttonExit As New Windows.Forms.Button With _
       {.Size = New Drawing.Size(80, 30), _
        .Location = New Drawing.Point(5, 5), _
        .Text = "Exit", .TextAlign = ContentAlignment.MiddleCenter}

    Public Sub buttonExit_Click() Handles buttonExit.MouseClick
        Application.Exit()
    End Sub

    Public WithEvents buttonRun As New Windows.Forms.Button With _
        {.Size = New Drawing.Size(80, 30), _
         .Location = New Drawing.Point(90, 5), _
         .Text = "Run", .TextAlign = ContentAlignment.MiddleCenter}

    Public Sub buttonRun_Click() Handles buttonRun.MouseClick
        MainProgram()
    End Sub

    Public GroupboxEdit As New Windows.Forms.GroupBox With _
        {.Size = New Drawing.Size(485, 325), _
         .Location = New Drawing.Point(5, 40)}

    Public DisplayText As New Windows.Forms.Label With _
        {.Size = New Drawing.Size(300, 250), _
         .Location = New Drawing.Point(5, 450), _
         .Text = "", .TextAlign = ContentAlignment.TopLeft}
```

```
'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////
'// Main program.
'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////

Public Sub MainProgram()
    '
    ' Read the parameters from groupboxEdit.
    ReadParameters()
    '
    ' Build the Joukowski airfoil.
    DisplayText.Text = "Now building Joukowski airfoil ..." & vbCrLf
    DisplayText.Refresh()
    JOUK_Airfoil()
    '
    ' Align the Joukowski airfoil.
    DisplayText.Text = "Now aligning the airfoil ..."
    DisplayText.Refresh()
    JOUK_Align()
    '
    ' Determine the surface points to send to GMesh.
    DisplayText.Text = "Now discretizing the airfoil for GMesh ..."
    DisplayText.Refresh()
    JOUK_Discretize()
    '
    ' Rotate the airfoil to the angle of attack.
    DisplayText.Text = "Now rotating to the angle of attack ..."
    DisplayText.Refresh()
    JOUK_Rotate()
    '
    ' Scale the distances expressed in chords into meters
    DistanceIm = -DistanceIc * JOUKchord
    DistanceTm = DistanceTc * JOUKchord
    DistanceRm = DistanceRc * JOUKchord
    DistanceOm = -DistanceOc * JOUKchord
    '
    ' Now building the output file to send to GMesh.
    DisplayText.Text = "Now building the output file ..."
    DisplayText.Refresh()
    BuildGMeshFile()
    '
    ' All finished.
    DisplayText.Text = "All finished."
    DisplayText.Refresh()
End Sub

'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////
'// Subroutine to read date from groupboxEdit.
'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////

Public Sub ReadParameters()
    JOUKR = Val(tbR.Text)
    JOUKf = Val(tbF.Text)
    JOUKg = Val(tbG.Text)
    JOUKb = Val(tbB.Text)
    NumUpper = CInt(Val(tbNumU.Text))
    NumLower = CInt(Val(tbNumL.Text))
    AngleAttack = Val(tbAA.Text)
```

```
    OutputFileName = "Joukowski_" & Trim(Str(AngleAttack)) & "deg.geo.txt"
    DistanceIc = Val(tbDisF.Text)
    DistanceTc = Val(tbDisT.Text)
    DistanceRc = Val(tbDisR.Text)
    DistanceOc = Val(tbDisB.Text)
    CharLenUm = Val(tbCharLenU.Text)
    CharLenLm = Val(tbCharLenL.Text)
    CharLenULm = Val(tbCharLenUL.Text)
    CharLenLLm = Val(tbCharLenLL.Text)
    CharLenLRm = Val(tbCharLenLR.Text)
    CharLenURm = Val(tbCharLenUR.Text)
End Sub

'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////
'// Controls to enter data.  Note that there are four blocks, or Groups, of data.
'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////

Public RowGroup1 As Int32 = 10
Public RowGroup2 As Int32 = RowGroup1 + 135
Public RowGroup3 As Int32 = RowGroup1
Public RowGroup4 As Int32 = RowGroup1 + 135
Public ColGroup1 As Int32 = 10
Public ColGroup2 As Int32 = ColGroup1
Public ColGroup3 As Int32 = ColGroup1 + 270
Public ColGroup4 As Int32 = ColGroup1 + 270
Public TitleW As Int32 = 180
Public LabelCol As Int32 = 5
Public LabelW As Int32 = 130
Public ValueCol As Int32 = 135
Public ValueW As Int32 = 60

Public labelGroup1 As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(TitleW, 20), _
    .Location = New Drawing.Point(ColGroup1, RowGroup1), _
    .Text = "Joukowski airfoil parameters (meters)", _
    .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public labelR As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup1 + LabelCol, RowGroup1 + 25), _
    .Text = "Joukowski R", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbR As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup1 + ValueCol, RowGroup1 + 25), _
    .Text = Trim(Str(JOUKR)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelF As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup1 + LabelCol, RowGroup1 + 50), _
    .Text = "Joukouski f", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbF As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup1 + ValueCol, RowGroup1 + 50), _
```

```
    .Text = Trim(Str(JOUKf)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelG As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup1 + LabelCol, RowGroup1 + 75), _
    .Text = "Joukowski g", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbG As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup1 + ValueCol, RowGroup1 + 75), _
    .Text = Trim(Str(JOUKg)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelB As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup1 + LabelCol, RowGroup1 + 100), _
    .Text = "Joukowski b", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbB As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup1 + ValueCol, RowGroup1 + 100), _
    .Text = Trim(Str(JOUKb)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelGroup2 As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(TitleW, 20), _
    .Location = New Drawing.Point(ColGroup2, RowGroup2), _
    .Text = "Discretization of the Joukowski airfoil", _
    .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public labelNumU As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup2 + LabelCol, RowGroup2 + 25), _
    .Text = "Number panels on top", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbNumU As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup2 + ValueCol, RowGroup2 + 25), _
    .Text = Trim(Str(NumUpper)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelSpreadU As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup2 + LabelCol, RowGroup2 + 50), _
    .Text = "Panel spread on top", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public WithEvents cbSpreadUSine As New Windows.Forms.CheckBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup2 + ValueCol, RowGroup2 + 50), _
    .Text = "Sine", .TextAlign = ContentAlignment.MiddleCenter, _
    .Checked = True, _
    .Parent = GroupboxEdit}

Public WithEvents cbSpreadUEqual As New Windows.Forms.CheckBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
```

```
    .Location = New Drawing.Point(ColGroup2 + ValueCol + 65, RowGroup2 + 50), _
    .Text = "Equal", .TextAlign = ContentAlignment.MiddleCenter, _
    .Checked = False, _
    .Parent = GroupboxEdit}

Public labelNumL As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup2 + LabelCol, RowGroup2 + 75), _
    .Text = "Number panels on bottom", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbNumL As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup2 + ValueCol, RowGroup2 + 75), _
    .Text = Trim(Str(NumLower)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelSpreadL As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup2 + LabelCol, RowGroup2 + 100), _
    .Text = "Panel spread on bottom", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public WithEvents cbSpreadLSine As New Windows.Forms.CheckBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup2 + ValueCol, RowGroup2 + 100), _
    .Text = "Sine", .TextAlign = ContentAlignment.MiddleCenter, _
    .Checked = True, _
    .Parent = GroupboxEdit}

Public WithEvents cbSpreadLEqual As New Windows.Forms.CheckBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup2 + ValueCol + 65, RowGroup2 + 100), _
    .Text = "Equal", .TextAlign = ContentAlignment.MiddleCenter, _
    .Checked = False, _
    .Parent = GroupboxEdit}

Public labelAA As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup1 + LabelCol, RowGroup2 + 150), _
    .Text = "Angle of attack (deg)", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbAA As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup1 + ValueCol, RowGroup2 + 150), _
    .Text = Trim(Str(AngleAttack)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelGroup3 As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(TitleW, 20), _
    .Location = New Drawing.Point(ColGroup3, RowGroup3), _
    .Text = "Distance to boundary (chords)", _
    .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public labelDisF As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup3 + LabelCol, RowGroup3 + 25), _
    .Text = "Upstream boundary", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}
```

```
Public tbDisF As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup3 + ValueCol, RowGroup3 + 25), _
    .Text = Trim(Str(DistanceIc)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelDisT As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup3 + LabelCol, RowGroup3 + 50), _
    .Text = "Top boundary", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbDisT As New Windows.Forms.TextBox With _
    {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup3 + ValueCol, RowGroup3 + 50), _
    .Text = Trim(Str(DistanceTc)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelDisR As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup3 + LabelCol, RowGroup3 + 75), _
    .Text = "Downstream boundary", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbDisR As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup3 + ValueCol, RowGroup3 + 75), _
    .Text = Trim(Str(DistanceRc)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelDisB As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup3 + LabelCol, RowGroup3 + 100), _
    .Text = "Bottom boundary", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbDisB As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup3 + ValueCol, RowGroup3 + 100), _
    .Text = Trim(Str(DistanceOc)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelGroup4 As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(TitleW, 20), _
    .Location = New Drawing.Point(ColGroup4, RowGroup4), _
    .Text = "Characteristic lengths (meters)", _
    .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public labelCharLenU As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup4 + LabelCol, RowGroup4 + 25), _
    .Text = "Upper surface", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbCharLenU As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup4 + ValueCol, RowGroup4 + 25), _
    .Text = Trim(Str(CharLenUm)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}
```

```
Public labelCharLenL As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup4 + LabelCol, RowGroup4 + 50), _
    .Text = "Lower surface", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbCharLenL As New Windows.Forms.TextBox With _
    {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup4 + ValueCol, RowGroup4 + 50), _
    .Text = Trim(Str(CharLenLm)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelCharLenUL As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup4 + LabelCol, RowGroup4 + 75), _
    .Text = "Upper left", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbCharLenUL As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup4 + ValueCol, RowGroup4 + 75), _
    .Text = Trim(Str(CharLenULm)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelCharLenLL As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup4 + LabelCol, RowGroup4 + 100), _
    .Text = "Lower left", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbCharLenLL As New Windows.Forms.TextBox With _
    {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup4 + ValueCol, RowGroup4 + 100), _
    .Text = Trim(Str(CharLenLLm)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelCharLenLR As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup4 + LabelCol, RowGroup4 + 125), _
    .Text = "Lower right", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbCharLenLR As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup4 + ValueCol, RowGroup4 + 125), _
    .Text = Trim(Str(CharLenLRm)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}

Public labelCharLenUR As New Windows.Forms.Label With _
   {.Size = New Drawing.Size(LabelW, 20), _
    .Location = New Drawing.Point(ColGroup4 + LabelCol, RowGroup4 + 150), _
    .Text = "Upper right", .TextAlign = ContentAlignment.MiddleLeft, _
    .Parent = GroupboxEdit}

Public tbCharLenUR As New Windows.Forms.TextBox With _
   {.Size = New Drawing.Size(ValueW, 20), _
    .Location = New Drawing.Point(ColGroup4 + ValueCol, RowGroup4 + 150), _
    .Text = Trim(Str(CharLenURm)), .TextAlign = HorizontalAlignment.Left, _
    .Parent = GroupboxEdit}
```

```
    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////
    '// Handlers for the four checkboxes.
    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////

    Public Sub cbSpreadUSine_Click() Handles cbSpreadUSine.MouseClick
        If (cbSpreadUSine.Checked = True) Then
            cbSpreadUEqual.Checked = False
        Else
            cbSpreadUEqual.Checked = True
        End If
    End Sub

    Public Sub cbSpreadUEqual_Click() Handles cbSpreadUEqual.MouseClick
        If (cbSpreadUEqual.Checked = True) Then
            cbSpreadUSine.Checked = False
        Else
            cbSpreadUSine.Checked = True
        End If
    End Sub

    Public Sub cbSpreadLSine_Click() Handles cbSpreadLSine.MouseClick
        If (cbSpreadLSine.Checked = True) Then
            cbSpreadLEqual.Checked = False
        Else
            cbSpreadLEqual.Checked = True
        End If
    End Sub

    Public Sub cbSpreadLEqual_Click() Handles cbSpreadLEqual.MouseClick
        If (cbSpreadLEqual.Checked = True) Then
            cbSpreadLSine.Checked = False
        Else
            cbSpreadLSine.Checked = True
        End If
    End Sub

End Class
```

Listing of module Joukowski

```
Option Strict On
Option Explicit On

Public Module Joukowski

    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////
    '
    ' Sub JOUK_Airfoil() discretizes a Joukowski airfoil into physical co-ordinates in
    ' the p-q plane.
    '
    ' The original circle is divided into NumJOUK equal segments so the circumference
    ' will be characterized by NumJOUK + 1 points.  The p-q co-ordinates of all such
    ' points are stored in the two vectors: JOUKP(NumJOUK + 1) and JOUKQ(NumJOUK + 1).
    '
    ' The p-q co-ordinates are not adjusted, but are left with the values calculated by
    ' using the Joukowski transformation.
    '
    ' The (p,q) points are ordered starting at theta=0 and increasing to theta=360.
    '
    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////

    Public Sub JOUK_Airfoil()
        ' Local variables.
        Dim Theta As Double     ' an angle around the original circle
        Dim Xo As Double        ' x co-ordinate of a point on the original circle
        Dim Yo As Double        ' y co-ordinate of a point on the original circle
        Dim Xs As Double        ' x co-ordinate of a point on the shifted circle
        Dim Ys As Double        ' y co-ordinate of a point on the shifted circle
        Dim delTheta As Double = 2 * Math.PI / NumJOUK
        Dim SinTheta As Double
        Dim CosTheta As Double
        Dim PQrad As Double
        ' Main loop to step through the NumJOUK + 1 points on the original circle.
        For Itheta As Int32 = 1 To (NumJOUK + 1) Step 1
            Theta = (Itheta - 1) * delTheta
            SinTheta = Math.Sin(Theta)
            CosTheta = Math.Cos(Theta)
            Xo = JOUKR * CosTheta
            Yo = JOUKR * SinTheta
            Xs = Xo - JOUKf
            Ys = Yo + JOUKg
            PQrad = (Xs * Xs) + (Ys * Ys)
            JOUKP(Itheta) = Xs + (JOUKb * JOUKb * Xs / PQrad)
            JOUKQ(Itheta) = Ys - (JOUKb * JOUKb * Ys / PQrad)
        Next Itheta

        'Dim DS As String = "Airfoil near theta = 0"
        'For I = 1 To 10 Step 1
        '    DS = DS & vbCrLf & "I=" & Str(I) & "     " & _
        '        "Theta=" & Str((I - 1) * delTheta) & "     " & _
        '        "JOUKP=" & Str(JOUKP(I)) & "     " & _
        '        "JUOKQ=" & Str(JOUKQ(I))
        'Next
        'DS = DS & vbCrLf & "Airfoil near theta = 360"
        'For I = (NumJOUK + 1) To (NumJOUK - 200) Step -5
        '    DS = DS & vbCrLf & "I=" & Str(I) & "     " & _
        '        "Theta=" & Str((I - 1) * delTheta) & "     " & _
```

```
    '        "JOUKP=" & Str(JOUKP(I)) & "     " & _
    '        "JUOKQ=" & Str(JOUKQ(I))
    'Next
    'MsgBox(DS)

End Sub

'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////
'
' Sub JOUK_Align() translates and rotates the Joukowski airfoil so that the origin
' of the p-q frame of reference is at the leading edge and so that the p-axis passes
' through the trailing edge.
'
' The leading and trailing edges are found by a brute force search through all
' points, looking for those with the extreme horizontal values.
'
' The Itheta indices of the LE and TE are stored in JOUKLEIndex and JOUKTEIndex,
' respectively.
'
'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////

Public Sub JOUK_Align()
    Dim LEIndex As Int32            ' Index of the LE in the list of points
    Dim TEIndex As Int32            ' Index of the TE in the list of points
    Dim Ple As Double               ' p-q co-ordinates of the leading edge
    Dim Qle As Double
    Dim Pte As Double               ' p-q co-ordinates of the trailing edge
    Dim Qte As Double
    Dim Psi As Double               ' The necessary rotation angle
    Dim Prot(NumJOUK + 1) As Double ' Points rotated to level the chord line
    Dim Qrot(NumJOUK + 1) As Double
    Dim Pref(NumJOUK + 1) As Double ' Points on the horizontal line
    Dim Qref(NumJOUK + 1) As Double
    '
    ' Step #1: Determine the extreme points in the horizontal.
    Dim JOUKPmin As Double = +1.0E+25
    Dim JOUKPmax As Double = -1.0E+25
    For Itheta As Int32 = 1 To (NumJOUK + 1) Step 1
        If (JOUKP(Itheta) < JOUKPmin) Then
            JOUKPmin = JOUKP(Itheta)
            LEIndex = Itheta
        End If
        If (JOUKP(Itheta) > JOUKPmax) Then
            JOUKPmax = JOUKP(Itheta)
            TEIndex = Itheta
        End If
    Next Itheta

    'Dim DS As String
    'DS = "LEIndex=" & Str(LEIndex) & vbCrLf & _
    '    "JKOUKP(-1, 0, 1)=" & _
    '    Str(JOUKP(LEIndex - 1)) & " " & _
    '    Str(JOUKP(LEIndex)) & " " & _
    '    Str(JOUKP(LEIndex + 1)) & vbCrLf & _
    '    "JKOUKQ(-1, 0, 1)=" & _
    '    Str(JOUKQ(LEIndex - 1)) & " " & _
    '    Str(JOUKQ(LEIndex)) & " " & _
    '    Str(JOUKQ(LEIndex + 1)) & vbCrLf & _
    '    "TEIndex=" & Str(TEIndex) & vbCrLf & _
```

```
    '    "JKOUKP(-1, 0, 1)=" & _
    '    Str(JOUKP(TEIndex - 1)) & " " & _
    '    Str(JOUKP(TEIndex)) & " " & _
    '    Str(JOUKP(TEIndex + 1)) & vbCrLf & _
    '    "JKOUKQ(-1, 0, 1)=" & _
    '    Str(JOUKQ(TEIndex - 1)) & " " & _
    '    Str(JOUKQ(TEIndex)) & " " & _
    '    Str(JOUKQ(TEIndex + 1))
    'MsgBox(DS)

    '
    ' Step #2: Translate the airfoil to the leading edge.
    For Itheta = 1 To (NumJOUK + 1) Step 1
        Prot(Itheta) = JOUKP(Itheta) - JOUKPmin
        Qrot(Itheta) = JOUKQ(Itheta)
    Next Itheta
    '
    ' Step #3: Set the leading and trailing edge variables.
    Ple = Prot(LEIndex)
    Qle = Prot(LEIndex)
    Pte = Prot(TEIndex)
    Qte = Qrot(TEIndex)
    '
    ' Step #4: Determine the angle of rotation.
    Psi = Math.Atan2(Qte - Qle, Pte - Ple)
    '
    ' Step #5: Rotate the airfoil into a horizontal position.
    Dim SinPsi As Double = Math.Sin(Psi)
    Dim CosPsi As Double = Math.Cos(Psi)
    Dim Temp As Double
    For Itheta As Int32 = 1 To (NumJOUK + 1) Step 1
        Temp = (CosPsi * Prot(Itheta)) + (SinPsi * Qrot(Itheta))
        Qrot(Itheta) = (-SinPsi * Prot(Itheta)) + (CosPsi * Qrot(Itheta))
        Prot(Itheta) = Temp
    Next Itheta
    '
    ' Update certain global variables.
    For Itheta As Int32 = 1 To (NumJOUK + 1) Step 1
        JOUKP(Itheta) = Prot(Itheta)
        JOUKQ(Itheta) = Qrot(Itheta)
    Next Itheta
    JOUKLEIndex = LEIndex
    JOUKTEIndex = TEIndex
    JOUKchord = Prot(TEIndex) - Prot(LEIndex)
End Sub

'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////
'
' Sub JOUK_Discretize() determines the points on the surface which are closest to the
' user's selection for the the number and spacing of the points to be sent to GMesh.
' The points for the upper and lower surfaces are handled separately, so that
' different characteristic lengths can be applied.
'
' The points with the desired spacing are determined by dividing the chord up into
' either equal spaces or sinusoidally-varying spaces.  In both cases, the points are
' ordered from the leading edge to the trailing edge, separately, on both surfaces.
'
' It is important to note that the final co-ordinates PdesiredU and Qdesired are in
' the order from LE to TE, not by angle theta.
'
```

```
'////////////////////////////////////////////////////////////////////////////////
'////////////////////////////////////////////////////////////////////////////////

Public Sub JOUK_Discretize()
    '
    ' Step #1: Build a set of desired abscissas for the upper surface.
    If (Main.cbSpreadUEqual.Checked = True) Then
        '
        ' Case #1: Equal spacing.
        Dim delPU As Double = JOUKchord / NumUpper
        For Ip As Int32 = 1 To (NumUpper + 1) Step 1
            PdesiredU(Ip) = Ip * delPU
        Next Ip
    Else
        '
        ' Case #2: Sinusoidal spacing.
        Dim DelThetaU As Double = Math.PI / NumUpper
        Dim TotalU As Double
        For Iu As Int32 = 1 To NumUpper Step 1
            TotalU = TotalU + Math.Sin((Iu - 0.5) * DelThetaU)
        Next Iu
        PdesiredU(1) = 0
        For Iu As Int32 = 1 To NumUpper Step 1
            PdesiredU(Iu + 1) = PdesiredU(Iu) + _
                (Math.Sin((Iu - 0.5) * DelThetaU) / TotalU)
        Next Iu
        For Iu As Int32 = 1 To (NumUpper + 1) Step 1
            PdesiredU(Iu) = PdesiredU(Iu) * JOUKchord
        Next Iu
    End If

    'Dim DS As String = "Upper surface PdesiredU's"
    'For I As Int32 = 1 To 10 Step 1
    '    DS = DS & vbCrLf & Str(I) & Str(PdesiredU(I))
    'Next
    'For I As Int32 = 10 To 1 Step -1
    '    Dim J As Int32
    '    J = NumUpper + 2 - I
    '    DS = DS & vbCrLf & Str(J) & Str(PdesiredU(J))
    'Next
    'MsgBox(DS)

    '
    ' Step #2: Build a set of desired abscissas for the lower surface.
    If (Main.cbSpreadLEqual.Checked = True) Then
        '
        ' Case #1: Equal spacing.
        Dim delPL As Double = JOUKchord / NumLower
        For Ip As Int32 = 1 To (NumLower + 1) Step 1
            PdesiredL(Ip) = Ip * delPL
        Next Ip
    Else
        '
        ' Case #2: Sinusoidal spacing.
        Dim DelThetaL As Double = Math.PI / NumLower
        Dim TotalL As Double
        For Il As Int32 = 1 To NumLower Step 1
            TotalL = TotalL + Math.Sin((Il - 0.5) * DelThetaL)
        Next Il
        PdesiredL(1) = 0
        For Il As Int32 = 1 To NumLower Step 1
```

```
        PdesiredL(Il + 1) = PdesiredL(Il) + _
            (Math.Sin((Il - 0.5) * DelThetaL) / TotalL)
    Next Il
    For Il As Int32 = 1 To (NumLower + 1) Step 1
        PdesiredL(Il) = PdesiredL(Il) * JOUKchord
    Next Il

    'DS = "Lower surface PdesiredL's"
    'For I As Int32 = 1 To 10 Step 1
    '    DS = DS & vbCrLf & Str(I) & Str(PdesiredL(I))
    'Next
    'For I As Int32 = 10 To 1 Step -1
    '    Dim J As Int32
    '    J = NumUpper + 2 - I
    '    DS = DS & vbCrLf & Str(J) & Str(PdesiredL(J))
    'Next
    'MsgBox(DS)

End If
'
' Step #3: Select the nearest points on the upper surface.  Note that the upper
' surface consist of angles in two ranges: (i) from theta=0 degrees, which is
' Index=1, to LEIndex, and (ii) from the trailing edge, where theta is just under
' 360 degrees, at TEIndex, to theta=360 degrees.
Dim ErrorU As Double
Dim IndexU As Int32
For Iu As Int32 = 2 To NumUpper Step 1
    ErrorU = +1.0E+25
    IndexU = -1
    For Itheta As Int32 = 1 To (NumJOUK + 1) Step 1
        If ((Itheta <= JOUKLEIndex) Or (Itheta >= JOUKTEIndex)) Then
            If ((Math.Abs(JOUKP(Itheta) - PdesiredU(Iu))) < ErrorU) Then
                ErrorU = Math.Abs(JOUKP(Itheta) - PdesiredU(Iu))
                IndexU = Itheta
            End If
        End If
    Next Itheta
    If (IndexU < 0) Then
        MsgBox("Error: Could not find nearest point on the upper surface.")
        Exit Sub
    End If
    PdesiredU(Iu) = JOUKP(IndexU)
    QdesiredU(Iu) = JOUKQ(IndexU)
Next Iu

'Dim DS As String
'DS = "Upper surface points"
'For I As Int32 = 1 To 10 Step 1
'    DS = DS & vbCrLf & Str(I) & " " & Str(PdesiredU(I)) & " " & Str(QdesiredU(I))
'Next
'For I As Int32 = 10 To 1 Step -1
'    Dim J As Int32
'    J = NumUpper + 2 - I
'    DS = DS & vbCrLf & Str(J) & " " & Str(PdesiredU(J)) & " " & Str(QdesiredU(J))
'Next
'MsgBox(DS)

'
' Step #4: Select the nearest points on the lower surface.  Note that the lower
' surface consists of points in a single range, from LEIndex to TEIndex.
Dim ErrorL As Double
```

```
        Dim IndexL As Int32
        For Il As Int32 = 2 To NumLower Step 1
            ErrorL = +1.0E+25
            IndexL = -1
            For Itheta As Int32 = 1 To (NumJOUK + 1) Step 1
                If ((Itheta >= JOUKLEIndex) And (Itheta <= JOUKTEIndex)) Then
                    If ((Math.Abs(JOUKP(Itheta) - PdesiredL(Il))) < ErrorL) Then
                        ErrorL = Math.Abs(JOUKP(Itheta) - PdesiredL(Il))
                        IndexL = Itheta
                    End If
                End If
            Next Itheta
            If (IndexL < 0) Then
                MsgBox("Error: Could not find nearest point on the lower surface.")
                Exit Sub
            End If
            PdesiredL(Il) = JOUKP(IndexL)
            QdesiredL(Il) = JOUKQ(IndexL)
        Next Il

        'Dim DS As String
        'DS = "Lower surface points"
        'For I As Int32 = 1 To 30 Step 1
        '    DS = DS & vbCrLf & Str(I) & " " & Str(PdesiredL(I)) & " " & Str(QdesiredL(I))
        'Next
        'For I As Int32 = 10 To 1 Step -1
        '    Dim J As Int32
        '    J = NumUpper + 2 - I
        '    DS = DS & vbCrLf & Str(J) & " " & Str(PdesiredL(J)) & " " & Str(QdesiredL(J))
        'Next
        'MsgBox(DS)
    End Sub

    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////
    '
    ' Sub JOUK_Rotate() rotates the points on the airfoil to the angle of attack.  The
    ' upper and lower surfaces keep their identities.
    '
    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////

    Public Sub JOUK_Rotate()
        Dim cosAA As Double = Math.Cos(AngleAttack * Math.PI / 180)
        Dim sinAA As Double = Math.Sin(AngleAttack * Math.PI / 180)
        Dim xValue As Double
        Dim yValue As Double
        For Iu As Int32 = 1 To (NumUpper + 1) Step 1
            xValue = (cosAA * PdesiredU(Iu)) + (sinAA * QdesiredU(Iu))
            yValue = (cosAA * QdesiredU(Iu)) - (sinAA * PdesiredU(Iu))
            PdesiredU(Iu) = xValue
            QdesiredU(Iu) = yValue
        Next Iu
        For Il As Int32 = 1 To (NumLower + 1) Step 1
            xValue = (cosAA * PdesiredL(Il)) + (sinAA * QdesiredL(Il))
            yValue = (cosAA * QdesiredL(Il)) - (sinAA * PdesiredL(Il))
            PdesiredL(Il) = xValue
            QdesiredL(Il) = yValue
        Next Il
    End Sub
End Module
```

Listing of module GMesh

```
Option Strict On
Option Explicit On

Public Module GMesh

    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////
    '
    ' Sub BuildGMeshFile() constructs the output file.
    '
    '////////////////////////////////////////////////////////////////////////////////
    '////////////////////////////////////////////////////////////////////////////////

    Public Sub BuildGMeshFile()
        '
        ' Step #1: Open the output file.
        Filewriter = New System.IO.StreamWriter(OutputFileName)
        '
        ' Step #2: Write a header to the output file with the airfoil parameters.
        Filewriter.Write("// Joukowski profile parameters:" & vbCrLf)
        Filewriter.Write("//   R = " & FormatNumber(JOUKR, 6, TriState.True) & _
            " meters" & vbCrLf)
        Filewriter.Write("//   f = " & FormatNumber(JOUKf, 6, TriState.True) & _
            " meters" & vbCrLf)
        Filewriter.Write("//   g = " & FormatNumber(JOUKg, 6, TriState.True) & _
            " meters" & vbCrLf)
        Filewriter.Write("//   b = " & FormatNumber(JOUKb, 6, TriState.True) & _
            " meters" & vbCrLf)
        Filewriter.Write("//   chord = " & _
            FormatNumber(JOUKchord, 4, TriState.True) & " meters" & vbCrLf)
        Filewriter.Write(vbCrLf)
        '
        ' Step #3: Write the angle of attack.
        Filewriter.Write("// Angle of attack = " & Trim(Str(AngleAttack)) & " degrees" & _
            vbCrLf & vbCrLf)
        '
        ' Step #4: Write the characteristic lengths.
        Filewriter.Write("// Characteristic lengths, in meters:" & vbCrLf)
        Filewriter.Write("lcUpperSurface = " & Trim(Str(CharLenUm)) & ";" & vbCrLf)
        Filewriter.Write("lcLowerSurface = " & Trim(Str(CharLenLm)) & ";" & vbCrLf)
        Filewriter.Write("lcUpperLeft = " & Trim(Str(CharLenULm)) & ";" & vbCrLf)
        Filewriter.Write("lcUpperRight = " & Trim(Str(CharLenURm)) & ";" & vbCrLf)
        Filewriter.Write("lcLowerRight = " & Trim(Str(CharLenLRm)) & ";" & vbCrLf)
        Filewriter.Write("lcLowerLeft = " & Trim(Str(CharLenLLm)) & ";" & vbCrLf)
        Filewriter.Write(vbCrLf)
        Filewriter.Write("// Use MeshAdapt algorithm for 2D" & vbCrLf)
        Filewriter.Write("// Use Frontal alorithm for 3D" & vbCrLf)
        Filewriter.Write("Mesh.Algorithm = 1;" & vbCrLf)
        Filewriter.Write("Mesh.Algorithm3D = 4;" & vbCrLf)
        Filewriter.Write(vbCrLf)
        '
        ' Step #5: Write the list of points on the surface, starting with the trailing
        ' edge and proceeding counter-clockwise around the entire airfoil.  Locate the
        ' cross-section at a z-value of minus one-half millimeter.  When the airfoil is
        ' extruded one millimeter in the z-direction, it will then be centered on the
        ' x-y plane.
        Filewriter.Write("// X Y Z co-ordinates of points on the surface:" & vbCrLf)
        Filewriter.Write("Geometry.Tolerance = 1e-10;" & vbCrLf)
        Filewriter.Write("Geometry.AutoCoherence = 1;" & vbCrLf)
```

```
Filewriter.Write(vbCrLf & "// Trailing edge:" & vbCrLf)
Dim LastPointNumber As Int32 = 1000
Filewriter.Write( _
    "Point(" & Trim(Str(LastPointNumber)) & ") = { " & _
    FormatNumber(PdesiredU(NumUpper + 1), 12, TriState.True) & " , " & _
    FormatNumber(QdesiredU(NumUpper + 1), 12, TriState.True) & " , " & _
    "-0.0005 , lcUpperSurface };" & vbCrLf)
Filewriter.Write(vbCrLf & "// Upper surface, proceeding forward:" & vbCrLf)
For Iu As Int32 = NumUpper To 2 Step -1
    LastPointNumber = LastPointNumber + 1
    Filewriter.Write( _
        "Point(" & Trim(Str(LastPointNumber)) & ") = { " & _
        FormatNumber(PdesiredU(Iu), 12, TriState.True) & " , " & _
        FormatNumber(QdesiredU(Iu), 12, TriState.True) & " , " & _
        "-0.0005 , lcUpperSurface };" & vbCrLf)
Next Iu
Filewriter.Write(vbCrLf & "// Leading edge:" & vbCrLf)
LastPointNumber = LastPointNumber + 1
Filewriter.Write( _
    "Point(" & Trim(Str(LastPointNumber)) & ") = { " & _
    FormatNumber(PdesiredU(1), 12, TriState.True) & " , " & _
    FormatNumber(QdesiredU(1), 12, TriState.True) & " , " & _
    "-0.0005 , lcUpperSurface };" & vbCrLf)
Filewriter.Write(vbCrLf & "// Lower surface, proceeding aft:" & vbCrLf)
For Il As Int32 = 2 To NumLower Step 1
    LastPointNumber = LastPointNumber + 1
    Filewriter.Write( _
        "Point(" & Trim(Str(LastPointNumber)) & ") = { " & _
        FormatNumber(PdesiredL(Il), 12, TriState.True) & " , " & _
        FormatNumber(QdesiredL(Il), 12, TriState.True) & " , " & _
        "-0.0005 , lcLowerSurface };" & vbCrLf)
Next Il
Filewriter.Write(vbCrLf)
'
' Step #6: Spline the perimeter of the airfoil, counter-clockwise from
' the trailing edge.
Filewriter.Write("// Spline around the airfoil." & vbCrLf)
Filewriter.Write("Spline(1000) = { 1000: " & _
    Trim(Str(LastPointNumber)) & ", 1000 };" & vbCrLf)
Filewriter.Write("Line Loop(1001) = { 1000 };" & vbCrLf)
Filewriter.Write(vbCrLf)
'
' Step #7: Specify the four corners of the boundary rectangle, starting with
' the upper-left corner and proceeding counter-clockwise.
Filewriter.Write("// Corners of the boundary rectangle:" & vbCrLf)
Dim FirstBoundaryPointNumber As Int32 = LastPointNumber + 1
Filewriter.Write( _
   "Point(" & Trim(Str(FirstBoundaryPointNumber)) & ") = { " & _
   FormatNumber(DistanceIm, 9, TriState.True) & " , " & _
   FormatNumber(DistanceTm, 9, TriState.True) & " , " & _
   "-0.0005 , lcUpperLeft };" & vbCrLf)
Filewriter.Write( _
    "Point(" & Trim(Str(FirstBoundaryPointNumber + 1)) & ") = { " & _
    FormatNumber(DistanceIm, 9, TriState.True) & " , " & _
    FormatNumber(DistanceOm, 9, TriState.True) & " , " & _
    "-0.0005 , lcLowerLeft };" & vbCrLf)
Filewriter.Write( _
    "Point(" & Trim(Str(FirstBoundaryPointNumber + 2)) & ") = { " & _
    FormatNumber(DistanceRm, 9, TriState.True) & " , " & _
    FormatNumber(DistanceOm, 9, TriState.True) & " , " & _
    "-0.0005 , lcLowerRight };" & vbCrLf)
```

```
Filewriter.Write( _
    "Point(" & Trim(Str(FirstBoundaryPointNumber + 3)) & ") = { " & _
    FormatNumber(DistanceRm, 9, TriState.True) & " , " & _
    FormatNumber(DistanceTm, 9, TriState.True) & " , " & _
    "-0.0005 , lcUpperRight };" & vbCrLf)
Filewriter.Write(vbCrLf)
'
' Step #8: Connect the boundary points with lines.
Filewriter.Write("// Edges of the boundary rectangle:" & vbCrLf)
Dim FirstBoundaryLineNumber As Int32 = 1
Filewriter.Write("Line(1002) = { " & _
    Trim(Str(FirstBoundaryPointNumber)) & " , " & _
    Trim(Str(FirstBoundaryPointNumber + 1)) & " };" & vbCrLf)
Filewriter.Write("Line(1003) = { " & _
    Trim(Str(FirstBoundaryPointNumber + 1)) & " , " & _
    Trim(Str(FirstBoundaryPointNumber + 2)) & " };" & vbCrLf)
Filewriter.Write("Line(1004) = { " & _
    Trim(Str(FirstBoundaryPointNumber + 2)) & " , " & _
    Trim(Str(FirstBoundaryPointNumber + 3)) & " };" & vbCrLf)
Filewriter.Write("Line(1005) = { " & _
    Trim(Str(FirstBoundaryPointNumber + 3)) & " , " & _
    Trim(Str(FirstBoundaryPointNumber)) & " };" & vbCrLf)
Filewriter.Write("Line Loop(1006) = { 1002 , 1003 , 1004 , 1005 };" & vbCrLf)
Filewriter.Write(vbCrLf)
'
' Step #9: Combine the two line loops into a surface.  The first Line Loop
' defines the outside boundary; the subsequent Line Loops are treated as holes.
Filewriter.Write("// Create a two-dimensional surface:" & vbCrLf)
Dim FirstSurfaceNumber As Int32 = 1
Filewriter.Write( _
    "Plane Surface(1007) = { 1006 , 1001 };" & vbCrLf)
Filewriter.Write(vbCrLf)
'
' Step #10: Extrude the surface for one tick in the z-direction.  By default, the
' first entry defines the "top" of the extruded entity and subsequent entries
' proceed towards the bottom.  The format used here applies produces a structured
' grids in the extruded direction.   Note that the displacement of the extrusion
' must be in the same order of magnitude as the mesh dimension or OpenFoam will
' complain about high aspect ratio prisms.
Filewriter.Write("// Extrude the surface one mm in the z-direction:" & vbCrLf)
Filewriter.Write("Extrude { 0 , 0 , 0.001 } {" & vbCrLf & _
    "  Surface{ 1007 };" & vbCrLf & _
    "  Layers{ 1 };" & vbCrLf & _
    "  Recombine;" & vbCrLf & _
    "}" & vbCrLf)
Filewriter.Write(vbCrLf)
'
' Step #11: Define physical surfaces.
Filewriter.Write("// Define the physical surfaces:" & vbCrLf)
Dim A As String
A = "// It is necessary to open this file with GMesh twice." & vbCrLf & _
    "// Prior to the first execution, remove the following" & vbCrLf & _
    "// lines from the .txt file.  At the end of its execution," & vbCrLf & _
    "// GMesh will print a list of the seven surfaces, which it" & vbCrLf & _
    "// will identify by number.  We need to know what those" & vbCrLf & _
    "// numbers are.  GMesh will have ordered the surfaces in" & vbCrLf & _
    "// the order: Front, Exit, Bottom, Airfoil, Back, Inlet" & vbCrLf & _
    "// and Top, although it does not refer to them using these" & vbCrLf & _
    "// names.  Once you know what the seven numbers are, the" & vbCrLf & _
    "// following lines should be added back to the .txt file." & vbCrLf & _
    "// If the numbers shown in the next few lines are not the" & vbCrLf & _
```

```
            "// same as the seven numbers GMesh has just given you," & vbCrLf & _
            "// then edit the text so that it shows the GMesh numbers." & vbCrLf & _
            "// You should then open the .txt file with GMesh a second" & vbCrLf & _
            "// time." & vbCrLf
        Filewriter.Write(A)
        Filewriter.Write("Physical Surface(""FrontWall"") = { 1034 };" & vbCrLf)
        Filewriter.Write("Physical Surface(""Outlet"") = { 1025 };" & vbCrLf)
        Filewriter.Write("Physical Surface(""Bottom"") = { 1021 };" & vbCrLf)
        Filewriter.Write("Physical Surface(""Airfoil"") = { 1033 };" & vbCrLf)
        Filewriter.Write("Physical Surface(""BackWall"") = { 1007 };" & vbCrLf)
        Filewriter.Write("Physical Surface(""Inlet"") = { 1017 };" & vbCrLf)
        Filewriter.Write("Physical Surface(""Top"") = { 1029 };" & vbCrLf)
        Filewriter.Write(vbCrLf)
        '
        ' Step #12: Define the physical volume.
        Filewriter.Write("// Define the physical volume:" & vbCrLf)
        Filewriter.Write("Physical Volume(""Internal"") = { 1 };" & vbCrLf)
        '
        ' Step #13: Close the outout file.
        Filewriter.Close()
    End Sub

End Module
```

Listing of module Variables

```
Option Strict On
Option Explicit On

Public Module Variables

    ' p-q co-ordinates of the Joukowski airfoil
    Public NumJOUK As Int32 = 1000000
    Public JOUKR As Double = 0.4051
    Public JOUKf As Double = 0.03069
    Public JOUKg As Double = 0.02032
    Public JOUKb As Double = 0.3672
    Public JOUKP(NumJOUK + 1) As Double         ' p and q for NumJOUK + 1 points
    Public JOUKQ(NumJOUK + 1) As Double

    ' Basic facts about a Joukowski airfoil
    Public JOUKLEIndex As Int32                 ' Index of the leading edge
    Public JOUKTEIndex As Int32                 ' Index of the trailing edge
    Public JOUKchord As Double

    ' Discretization of the airfoil
    Public NumUpper As Int32 = 1000             ' Number of panels on the upper surface
    Public NumLower As Int32 = 1000             ' Number of panels on the lower surface
    Public SpreadUpper As String                ' Spread of points on the upper surface
    Public SpreadLower As String                ' Spread of points on the lower surface
    Public PdesiredU(NumUpper + 1) As Double    ' Final points sent to GMesh
    Public QdesiredU(NumUpper + 1) As Double
    Public PdesiredL(NumLower + 1) As Double
    Public QdesiredL(NumLower + 1) As Double

    ' Angle of attack (degrees)
    Public AngleAttack As Double = 5

    ' Specification of the boundary
    Public DistanceIc As Double = 5             ' Distance to Inlet, chords
```

```
    Public DistanceTc As Double = 5             ' Distance to Top, chords
    Public DistanceRc As Double = 6             ' Distance to Outlet, chords
    Public DistanceOc As Double = 5.5           ' Distance to Bottom, chords
    Public DistanceIm As Double                 ' Distance to Inlet, meters
    Public DistanceTm As Double                 ' Distance to Top, meters
    Public DistanceRm As Double                 ' Distance to Outlet, meters
    Public DistanceOm As Double                 ' Distance to Bottom, meters

    ' Characteristic lengths
    Public CharLenUm As Double = 0.0002         ' CharLen on upper surface, meters
    Public CharLenLm As Double = 0.0002         ' CharLen on lower surface, meters
    Public CharLenULm As Double = 0.25          ' CharLen at upper left corner, meters
    Public CharLenLLm As Double = 0.25          ' CharLen at lower left corner, meters
    Public CharLenLRm As Double = 0.25          ' CharLen at lower right corner, meters
    Public CharLenURm As Double = 0.25          ' CharLen at upper right corner, meters

    ' Outfile text file
    Public OutputFileName As String = _
        "Joukowski_" & Trim(Str(AngleAttack)) & "deg.geo.txt"
    Public Filewriter As System.IO.StreamWriter

End Module
```

Appendix "B"

Excerpts from a GMesh instruction file

The following is a listing of the file "`Joukowski_5deg.geo.txt`", which describes the configuration required to mesh the case where the angle of attack is 5°. When GMesh processes this file, it will write an output file named "`Joukowski_5deg.geo.msh`". This .msh file can be understood by OpenFoam. Typing the command "`gmshToFoam Joukowski_5deg.geo.msh`" in a terminal window opened in the case directory will prompt OpenFoam to construct the files needed to run the case.

Certain repetitive sections have been deleted from the following listing.

```
// Joukowski profile parameters:
//   R = 0.405100 meters
//   f = 0.030690 meters
//   g = 0.020320 meters
//   b = 0.367200 meters
//   chord = 1.4796 meters

// Angle of attack = 5 degrees

// Characteristic lengths, in meters:
lcUpperSurface = .0002;
lcLowerSurface = .0002;
lcUpperLeft = .25;
lcUpperRight = .25;
lcLowerRight = .25;
lcLowerLeft = .25;

// Use MeshAdapt algorithm for 2D
// Use Frontal alorithm for 3D
Mesh.Algorithm = 1;
Mesh.Algorithm3D = 4;

// X Y Z co-ordinates of points on the surface:
Geometry.Tolerance = 1e-10;
Geometry.AutoCoherence = 1;

// Trailing edge:
Point(1000) = { 1.473964514698 , -0.128955185476 , -0.0005 , lcUpperSurface };

// Upper surface, proceeding forward:
Point(1001) = { 1.473964590959 , -0.128912375523 , -0.0005 , lcUpperSurface };
Point(1002) = { 1.473957457669 , -0.128868381302 , -0.0005 , lcUpperSurface };
Point(1003) = { 1.473943105021 , -0.128822919513 , -0.0005 , lcUpperSurface };
Point(1004) = { 1.473921545181 , -0.128776079507 , -0.0005 , lcUpperSurface };
Point(1005) = { 1.473892782787 , -0.128727857344 , -0.0005 , lcUpperSurface };
```

.. .. .. Definitions of Point(1006) through Point(1994) are not shown .. .. ..

```
Point(1995) = { 0.000354642700 , 0.003006202664 , -0.0005 , lcUpperSurface };
Point(1996) = { 0.000287376388 , 0.002614558768 , -0.0005 , lcUpperSurface };
Point(1997) = { 0.000227443525 , 0.002222886531 , -0.0005 , lcUpperSurface };
Point(1998) = { 0.000174939037 , 0.001831928283 , -0.0005 , lcUpperSurface };
Point(1999) = { 0.000129669140 , 0.001440225800 , -0.0005 , lcUpperSurface };

// Leading edge:
Point(2000) = { 0.000000000000 , 0.000000000000 , -0.0005 , lcUpperSurface };
```

```
// Lower surface, proceeding aft:
Point(2001) = { 0.000061262741 , 0.000658290747 , -0.0005 , lcLowerSurface };
Point(2002) = { 0.000038040761 , 0.000267342097 , -0.0005 , lcLowerSurface };
Point(2003) = { 0.000022120667 , -0.000124315776 , -0.0005 , lcLowerSurface };
Point(2004) = { 0.000013565028 , -0.000515207291 , -0.0005 , lcLowerSurface };
Point(2005) = { 0.000012340045 , -0.000906057241 , -0.0005 , lcLowerSurface };
```

.. .. .. Definitions of Point(2006) through Point (2994) are not shown .. .. ..

```
Point(2995) = { 1.473856010667 , -0.129148136788 , -0.0005 , lcLowerSurface };
Point(2996) = { 1.473892114066 , -0.129112278436 , -0.0005 , lcLowerSurface };
Point(2997) = { 1.473921065901 , -0.129074994853 , -0.0005 , lcLowerSurface };
Point(2998) = { 1.473942745644 , -0.129036442167 , -0.0005 , lcLowerSurface };
Point(2999) = { 1.473957227399 , -0.128996537467 , -0.0005 , lcLowerSurface };

// Spline around the airfoil.
Spline(1000) = { 1000: 2999, 1000 };
Line Loop(1001) = { 1000 };

// Corners of the boundary rectangle:
Point(3000) = { -7.397974098 , 7.397974098 , -0.0005 , lcUpperLeft };
Point(3001) = { -7.397974098 , -8.137771508 , -0.0005 , lcLowerLeft };
Point(3002) = { 8.877568918 , -8.137771508 , -0.0005 , lcLowerRight };
Point(3003) = { 8.877568918 , 7.397974098 , -0.0005 , lcUpperRight };

// Edges of the boundary rectangle:
Line(1002) = { 3000 , 3001 };
Line(1003) = { 3001 , 3002 };
Line(1004) = { 3002 , 3003 };
Line(1005) = { 3003 , 3000 };
Line Loop(1006) = { 1002 , 1003 , 1004 , 1005 };

// Create a two-dimensional surface:
Plane Surface(1007) = { 1006 , 1001 };

// Extrude the surface one mm in the z-direction:
Extrude { 0 , 0 , 0.001 } {
  Surface{ 1007 };
  Layers{ 1 };
  Recombine;
}

// Define the physical surfaces:
// It is necessary to open this file with GMesh twice.
// Prior to the first execution, remove the following
// lines from the .txt file.  At the end of its execution,
// GMesh will print a list of the seven surfaces, which it
// will identify by number.  We need to know what those
// numbers are.  GMesh will have ordered the surfaces in
// the order: Front, Exit, Bottom, Airfoil, Back, Inlet
// and Top, although it does not refer to them using these
// names.  Once you know what the seven numbers are, the
// following lines should be added back to the .txt file.
// If the numbers shown in the next few lines are not the
// same as the seven numbers GMesh has just given you,
// then edit the text so that it shows the GMesh numbers.
// You should then open the .txt file with GMesh a second
// time.
Physical Surface("FrontWall") = { 1034 };
Physical Surface("Outlet") = { 1025 };
```

```
Physical Surface("Bottom") = { 1021 };
Physical Surface("Airfoil") = { 1033 };
Physical Surface("BackWall") = { 1007 };
Physical Surface("Inlet") = { 1017 };
Physical Surface("Top") = { 1029 };

// Define the physical volume:
Physical Volume("Internal") = { 1 };
```

Appendix "C"

Listing of the `/constant/polyMesh/boundary` file for the base case

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       polyBoundaryMesh;
    location    "constant/polyMesh";
    object      boundary;
}
// * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * //

7
(
    BackWall
    {
        type            empty;
        nFaces          1185811;
        startFace       1771035;
    }
    FrontWall
    {
        type            empty;
        nFaces          1185811;
        startFace       2956846;
    }
    Inlet
    {
        type            patch;
        nFaces          63;
        startFace       4142657;
    }
    Bottom
    {
        type            symmetryPlane;
        nFaces          66;
        startFace       4142720;
    }
    Outlet
    {
        type            patch;
        nFaces          63;
        startFace       4142786;
    }
    Top
    {
        type            symmetryPlane;
        nFaces          66;
        startFace       4142849;
    }
    Airfoil
    {
        type            wall;
```

```
        nFaces          15105;
        startFace       4142915;
    }
)

// ************************************************************************* //
```

Appendix "D"

Listing of `constant/RASProperties` and `constant/transportProperties`

```
/*---------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       dictionary;
    location    "constant";
    object      RASProperties;
}
RASModel        SpalartAllmaras;
turbulence      on;
printCoeffs     on;
```

```
/*---------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       dictionary;
    location    "constant";
    object      transportProperties;
}

// U.S. Standard Atmosphere
// Altitude---   Density-----  Dynamic visc---  Kinematic visc-
//      0 feet   1.225 kg/m^3  1.789E-5 Ns/m^2  1.4604E-5 m^2/s
//  5,000        0.7364        1.628E-5         2.2108E-5
// 10,000        0.4135        1.458E-5         3.5260E-5
// 15,000        0.1948        1.422E-5         7.2998E-5

transportModel  Newtonian;
nu              nu [0 2 -1 0 0 0 0] 1.4604e-05;
rho             rho [ 1 -3 0 0 0 0 0 ] 1.225;

CrossPowerLawCoeffs
{
    nu0             nu0 [ 0 2 -1 0 0 0 0 ] 1e-06;
    nuInf           nuInf [ 0 2 -1 0 0 0 0 ] 1e-06;
    m               m [ 0 0 1 0 0 0 0 ] 1;
    n               n [ 0 0 0 0 0 0 0 ] 1;
}

BirdCarreauCoeffs
{
    nu0             nu0 [ 0 2 -1 0 0 0 0 ] 1e-06;
    nuInf           nuInf [ 0 2 -1 0 0 0 0 ] 1e-06;
```

```
    k               k [ 0 0 1 0 0 0 0 ] 0;
    n               n [ 0 0 0 0 0 0 0 ] 1;
}
```

Appendix “E”

Listing of the four files in subdirectory `system/`

Listing of `system/controlDict`

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       dictionary;
    object      controlDict;
}
application     simpleFoam;
startFrom       latestTime;
startTime       0;
stopAt          endTime;
endTime         100000;
deltaT          1;
writeControl    timeStep;
writeInterval   250;
purgeWrite      0;
writeFormat     ascii;
writePrecision  7;
writeCompression off;
timeFormat      general;
timePrecision   6;
runTimeModifiable true;
functions
{
  ForceOnAirfoil
  {
    type                forces;
    functionObjectLibs  ( "libforces.so" );
    patches             ( Airfoil );
    rhoName             rhoInf;
    pName               p;
    UName               U;
    log                 true;
    rhoInf              1.225;
    CofR                ( 0 0 0 );
    outputControl       timeStep;
    outputInterval      1;
  }
};
```

Listing of `system/decomposeParDict`

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       dictionary;
    object      decomposeParDict;
}
numberOfSubdomains  8;
method              scotch;
scotchCoeffs
{}
distributed         no;
roots
    ();

// To run a case in parallel, do this:
// 1.  <prompt> decomposePar
// 2.  <prompt> mpirun -np 8 simpleFoam -parallel | tee ofLog.txt
// 3.  When done, <prompt> reconstructPar
```

Listing of `system/fvSchemes`

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       dictionary;
    location    "system";
    object      fvSchemes;
}

ddtSchemes
{
    default         steadyState;
}

gradSchemes
{
    default         Gauss linear;
    grad(p)         Gauss linear;
    grad(U)         Gauss linear;
}

divSchemes
{
    default         none;
    div(phi,U)      Gauss linearUpwind grad(U);
    div(phi,nuTilda) Gauss linearUpwind grad(nuTilda);
    div((nuEff*dev(T(grad(U))))) Gauss linear;
}

laplacianSchemes
{
    default         none;
    laplacian(nuEff,U) Gauss linear corrected;
    laplacian((1|A(U)),p) Gauss linear corrected;
    laplacian(DnuTildaEff,nuTilda) Gauss linear corrected;
    laplacian(1,p)  Gauss linear corrected;
}

interpolationSchemes
{
    default         linear;
    interpolate(U)  linear;
}

snGradSchemes
{
    default         corrected;
}

fluxRequired
{
    default         no;
    p               ;
}
```

Listing of `system/fvSolution`

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       dictionary;
    location    "system";
    object      fvSolution;
}
solvers
{
    p
    {
        solver          GAMG;
        tolerance       1e-06;
        relTol          0.1;
        smoother        GaussSeidel;
        nPreSweeps      0;
        nPostSweeps     2;
        cacheAgglomeration true;
        nCellsInCoarsestLevel 10;
        agglomerator    faceAreaPair;
        mergeLevels     1;
    }
    U
    {
        solver          smoothSolver;
        smoother        GaussSeidel;
        nSweeps         2;
        tolerance       1e-08;
        relTol          0.1;
    }
    nuTilda
    {
        solver          smoothSolver;
        smoother        GaussSeidel;
        nSweeps         2;
        tolerance       1e-08;
        relTol          0.1;
    }
}
SIMPLE
{
    nNonOrthogonalCorrectors 0;
    pRefCell        0;
    pRefValue       0;
    residualControl
    {
        p               1e-5;
        U               1e-5;
        nuTilda         1e-5;
    }
}
relaxationFactors
{
```

```
    fields
    {
        p               0.35;
    }
    equations
    {
        U               0.65;
        nuTilda         0.8;
    }
}
```

Appendix “F”

Listing of the four files in subdirectory 0/

Listing of `0/p`

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       volScalarField;
    object      p;
}
dimensions      [0 2 -2 0 0 0 0];
internalField   uniform 0;
boundaryField
{
    Inlet
    {
        type            freestreamPressure;
    }
    Outlet
    {
        type            freestreamPressure;
    }
    RightWall
    {
        type            empty;
    }
    LeftWall
    {
        type            empty;
    }
    Top
    {
        type            symmetryPlane;
    }
    Bottom
    {
        type            symmetryPlane;
    }
    Airfoil
    {
        type            zeroGradient;
    }
}
```

Listing of `0/U`

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       volVectorField;
    object      U;
}

// 100mph = 44.704 m/s

dimensions      [0 1 -1 0 0 0 0];
internalField   uniform (44.704 0 0);

boundaryField
{
    Inlet
    {
        type            freestream;
        freestreamValue uniform (44.704 0 0);
    }
    Outlet
    {
        type            freestream;
        freestreamValue uniform (44.704 0 0);
    }
    RightWall
    {
        type            empty;
    }
    LeftWall
    {
        type            empty;
    }
    Top
    {
        type            symmetryPlane;
    }
    Bottom
    {
        type            symmetryPlane;
    }
    Airfoil
    {
        type            fixedValue;
        value           uniform (0 0 0);
    }
}
```

Listing of `0/nut`

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       volScalarField;
    object      nut;
}

// Set the freestream value of nut to one-tenth of nuTilda.
// If nuTilda = 0.274, then nut = 0.0274.
dimensions      [0 2 -1 0 0 0 0];
internalField   uniform 0.0274;
boundaryField
{
    Inlet
    {
        type            freestream;
        freestreamValue uniform 0.0274;
    }
    Outlet
    {
        type            freestream;
        freestreamValue uniform 0.0274;
    }
    RightWall
    {
        type            empty;
    }
    LeftWall
    {
        type            empty;
    }
    Top
    {
        type            symmetryPlane;
    }
    Bottom
    {
        type            symmetryPlane;
    }
    Airfoil
    {
        type            nutUSpaldingWallFunction;
        value           uniform 0;
    }
}
```

Listing of `0/nuTilda`

```
/*--------------------------------*- C++ -*----------------------------------*\
| =========                 |                                                 |
| \\      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
|  \\    /   O peration     | Version:  2.1.1                                 |
|   \\  /    A nd           | Web:      www.OpenFOAM.org                      |
|    \\/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version     2.0;
    format      ascii;
    class       volScalarField;
    object      nuTilda;
}

// Calculate nuTilda = sqrt(1.5) * UIl, where
//   U = 44.704 m/s
//   I = 0.05 is the estimated turbulent intensity
//   l = 10 centimeters is the estimated length scale
// Then, nuTilda = 0.274
// Set the freestream value of nuTilda to five times this.

dimensions      [0 2 -1 0 0 0 0];
internalField   uniform 1.37;
boundaryField
{
    Inlet
    {
        type            freestream;
        freestreamValue uniform 1.37;
    }
    Outlet
    {
        type            freestream;
        freestreamValue uniform 1.37;
    }
    RightWall
    {
        type            empty;
    }
    LeftWall
    {
        type            empty;
    }
    Top
    {
        type            symmetryPlane;
    }
    Bottom
    {
        type            symmetryPlane;
    }
    Airfoil
    {
        type            fixedValue;
        value           uniform 0;
    }
}
```