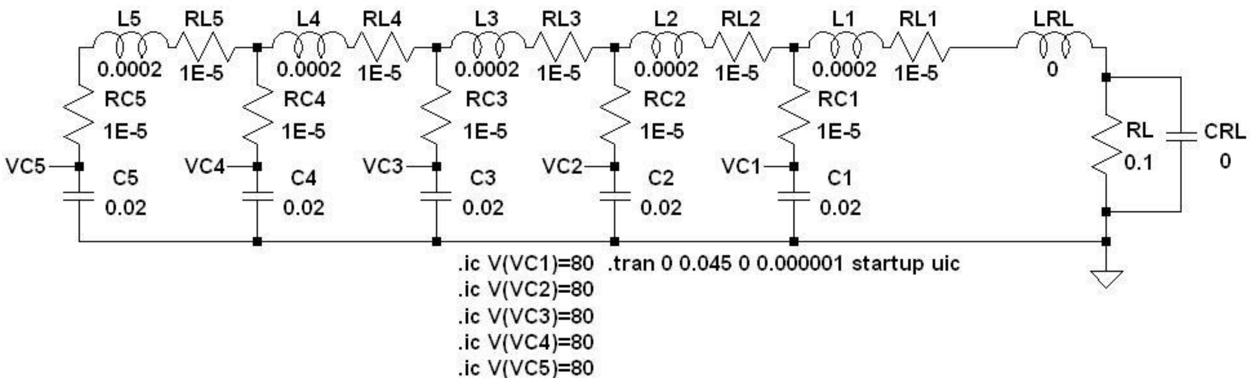
Rayleigh Pulse Forming Network

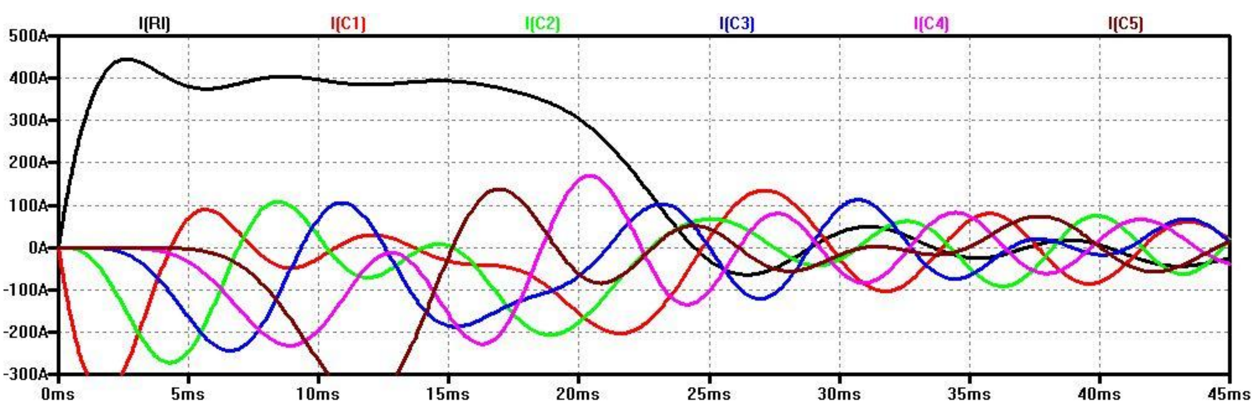
Part II – Assessment of sensitivity

The pulse forming networks we looked at in Part I of this paper were ideal. The capacitors and inductors did not suffer from any internal resistance and all were perfectly matched. In this part of the paper we will take a quick look at the consequences of non-ideality. The simplest way to do this is to simulate the network with one or more of the components changed in some way. For this purpose, a SPICE model was prepared and the simulations carried out using SPICE. A five-stage network was used as the basis for the assessment. It contains five capacitors with a value of $C = 0.0\ 2\text{F} = 20\ \text{mF}$ and five inductors with a nominal value of $L = 0.0002\ \text{H} = 200\ \text{mH}$. All capacitors were initially charged to $V(0) = 80\ V$. The load consisted of a small resistor $R_L = 0.1\ \Omega$. The nominal square wave produced by this network should be 400 A for a period of 20 ms.

The SPICE model is shown in the following figure.



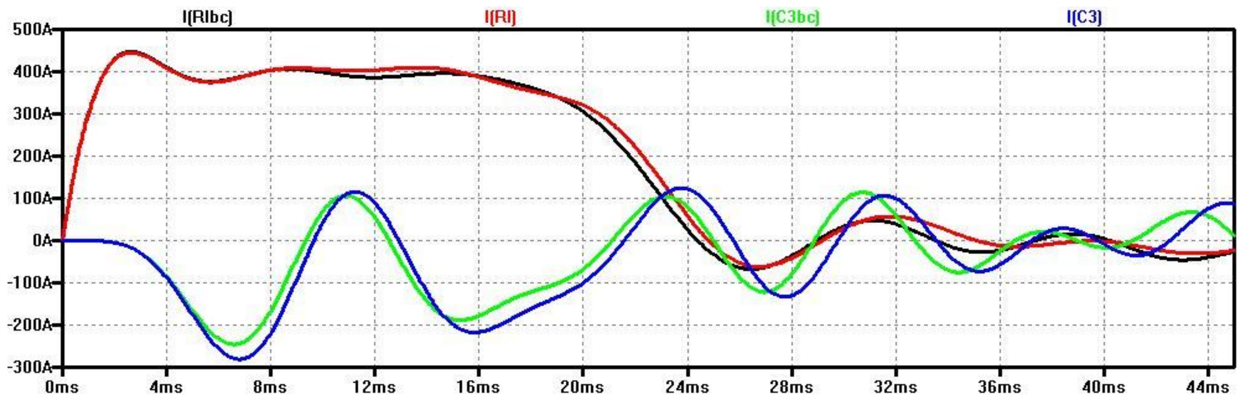
Provision has been made for the equivalent series resistance of the capacitors (resistors R_{C_1} through R_{C_5}) and for the Ohmics resistance of the windings of the indictors (resistors R_{L_1} through R_{L_5}). Provision has also been made by the inductance L_{R_L} and parallel capacitance C_{R_L} of the load resistor. My version of SPICE does not permit resistances which are identically equal to zero, so a very small resistance has been assigned where necessary. The waveforms of the currents through the five capacitors, and through the load resistor, are shown in the following graph.



The waveform in black is the current through the load resistors. It can be seen that the capacitors take turns powering the load, in the order C_1, C_2, C_3 and then C_5, C_4.

Sensitivity #1 – Increasing the capacitance of C_3 by 10%

Let us increase the capacitance of one of the capacitors by 10%, which is within the common range of variability. By making the change to C_3, which is in the middle of the network, we should be able to see the effect of the change both before and after the time when C_3 "comes on line". Note that increasing the capacitance will give C_3 more initial energy if all the capacitors are charged to the same voltage (a common arrangement). Since the capacitors as a group contain more energy, the current pulse through the load resistor should be greater or longer, or both. The following graph shows what happens.

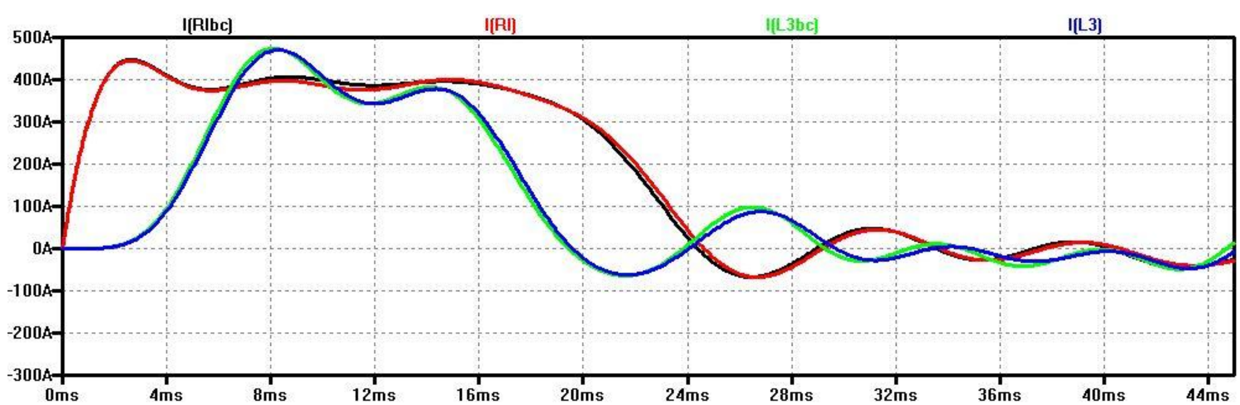


The waveform in black is the "base case" current through the load resistor, where the "base case" is the ideal configuration plotted above. The waveform in red is the current where C_3's capacitance is 0.022 F rather than 0.02 F. Indeed, the pulse is very slightly greater and slightly longer.

The waveforms shown in green (base case) and in blue (greater value for C_3) show the current delivered by C_3 in the two cases. C_3 comes on line stronger and a little bit later when it has increased capacitance.

Sensitivity #2 – Increasing the inductance of L_3 by 10%

In this variation, we will look at increasing the value of one of the inductors by 10%. This is a larger change than would normally be expected. In many circumstances, the inductors will be wound specifically for this device. They will likely be air-core coils wound with heavy wire and characterized by relatively few turns. The winding process would have to be very crude in order for the inductance of one of the inductors to differ from its counterparts by as much as 10%. As before, we will apply the change to the inductor in the middle of the network. The following graph shows what happens.

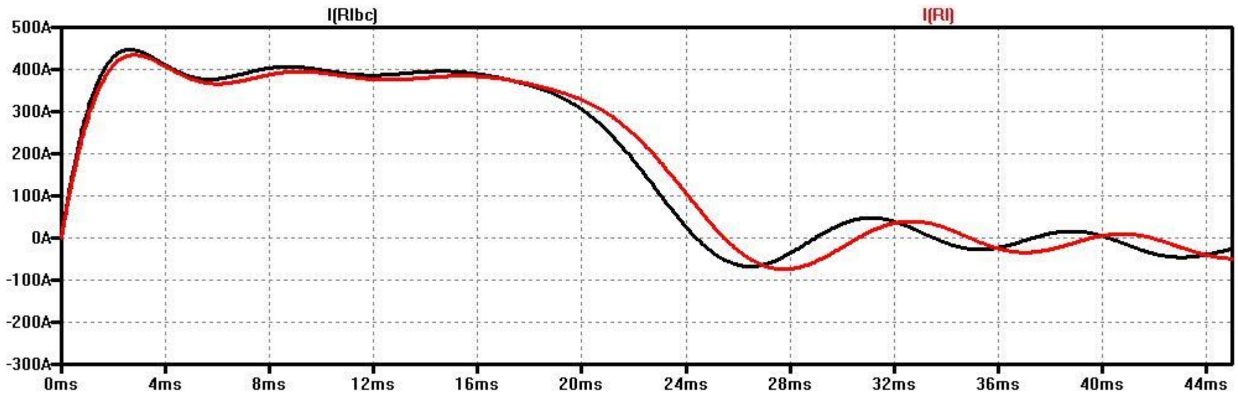


The waveform in black is the base case current through the load resistor. The waveform in red is the current where L_3's inductance is increased from 200 mH to 220 mH.

Since the inductors are not sources of energy in the circuit, but are temporary storage for energy, a change in an inductor should be energy "neutral" for the circuit. The same amount of energy will be delivered to the load in either case. We observe that the current is very slightly less and that the pulse is very slightly longer. The waveforms shown in green (base case) and in blue (greater value for L_3) show the current delivered by L_3 in the two cases. L_3 comes on line a little weaker and a little weaker when it has increased inductance.

Sensitivity #3 – Increasing all of the inductances by 10%

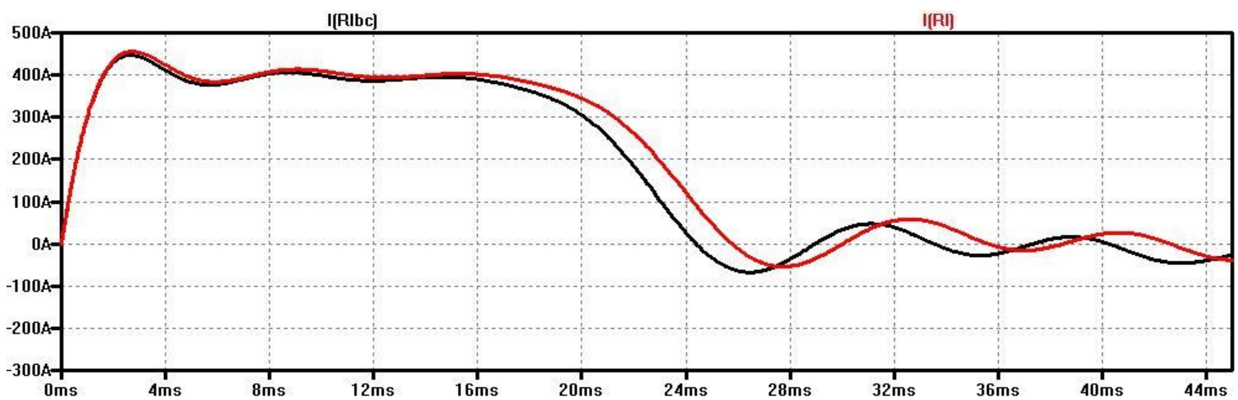
In this variation, we will look at a more likely case – that the inductors do not have the value that was planned for them when they were wound. The following graph shows what happens if all five inductors have a value of 220 mH rather than the design value of 200 mH.



The impact of the change on the load current is modest – the length of the pulse increases by about 1 ms, or approximately 4%. I am happily surprised. I had thought that a five-stage network needed to be well balanced or, stated differently, that it would be very sensitive to the component values. This does not seem to be the case.

Sensitivity #4 – Increasing all of the capacitances by 10%

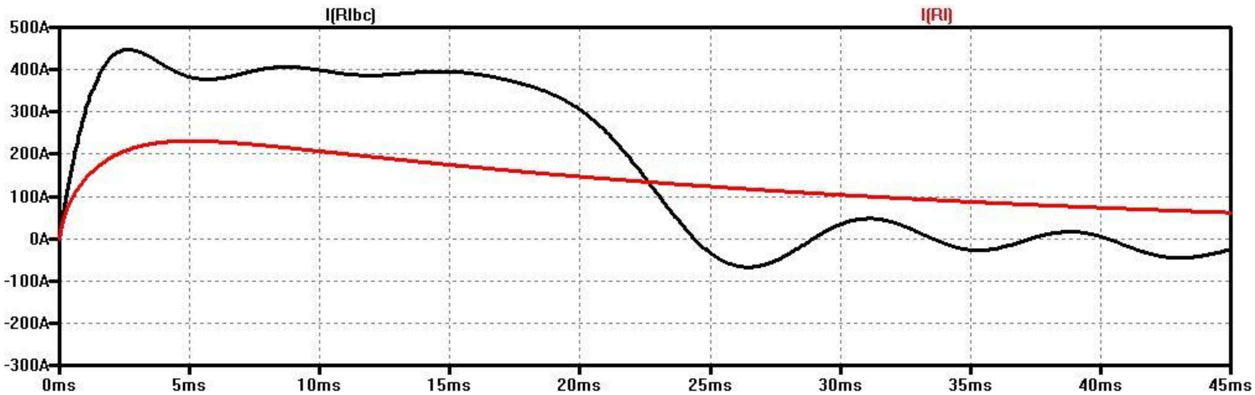
In this variation, we will examine increasing all five capacitors from 0.02 F to 0.022 F. look at a more likely case – that the inductors do not have the value that was planned for them when they were wound. The following graph shows



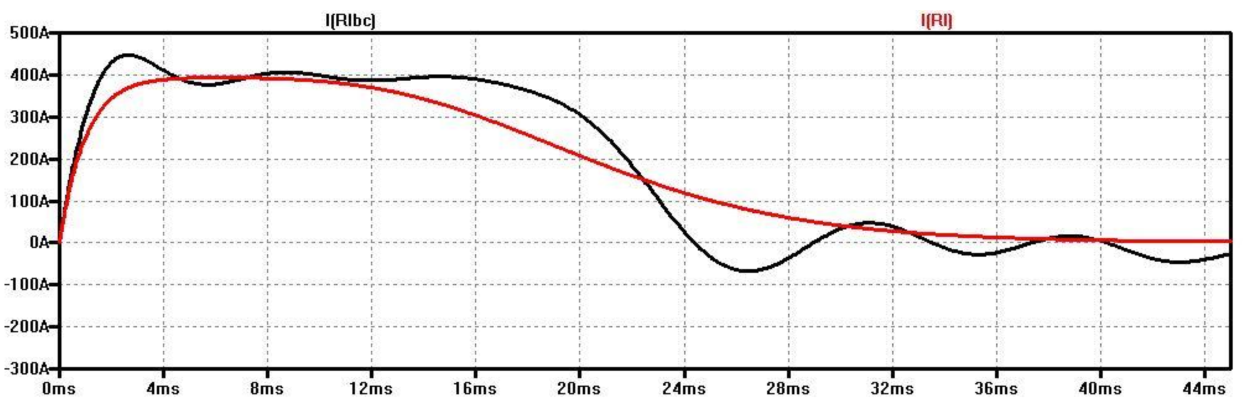
As before, the black waveform is the base case current through the load and the red curve is the load current with the revised component values. While the increase in the length of the pulse is greater for a ten percent change in the capacitances compared with a ten percent change in the inductances, it is still manageable. It should be observed that much of the extra energy is expended in the cycles which follow the main pulse.

Sensitivity #5 – Adding 1Ω ESR and 0.1Ω ESR to each capacitor

Arguably, the principal characteristic of a real capacitor, other than its capacitance, is its equivalent series resistance. For example, a "low-ESR" $2200\ \mu F$ capacitor will likely have an equivalent series resistance in the range from 0.01Ω to 0.1Ω. Wiring nine such capacitors in series to build a single 0.0198 F capacitor would have the effect of adding the resistances in series. The equivalent series resistance of the composite capacitor could be as high as 1Ω. This is ten times as large as the load resistance. Let us see what happens.



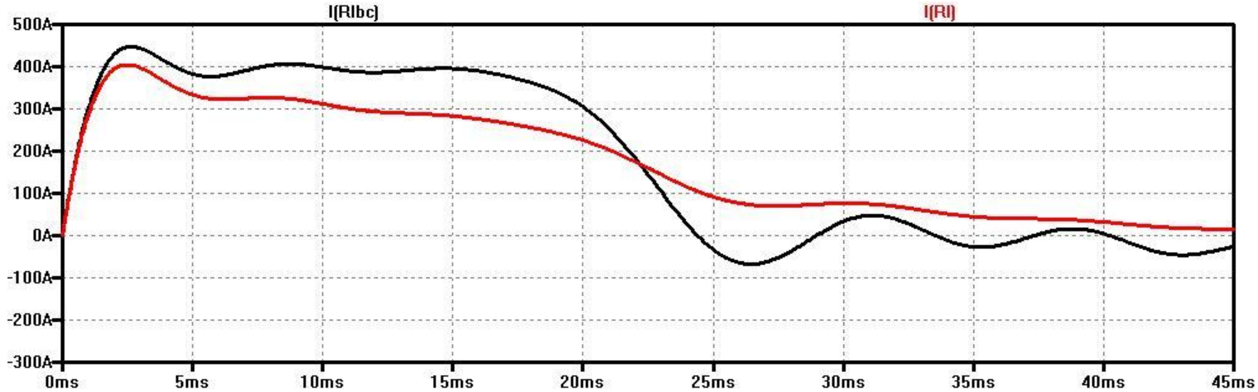
The effect of the equivalent series resistance is much more serious than the previous variations. Not much is left of the pulse. If the ESR per capacitor can be kept at the low end of the range – so that the ESR of the composite capacitor is 0.1Ω, just equal to the load resistance – then the waveforms look like the following.



The pulse in this graph is closer to the original one, and may be acceptable for some applications. The lesson here is that the equivalent series resistance of the load capacitors should be as low as possible, and at least less than the value of the load resistance, for the pulse forming network to be able to deliver a discernable pulse.

Sensitivity #6 – Adding 0.02Ω Ohmic resistance to each inductor

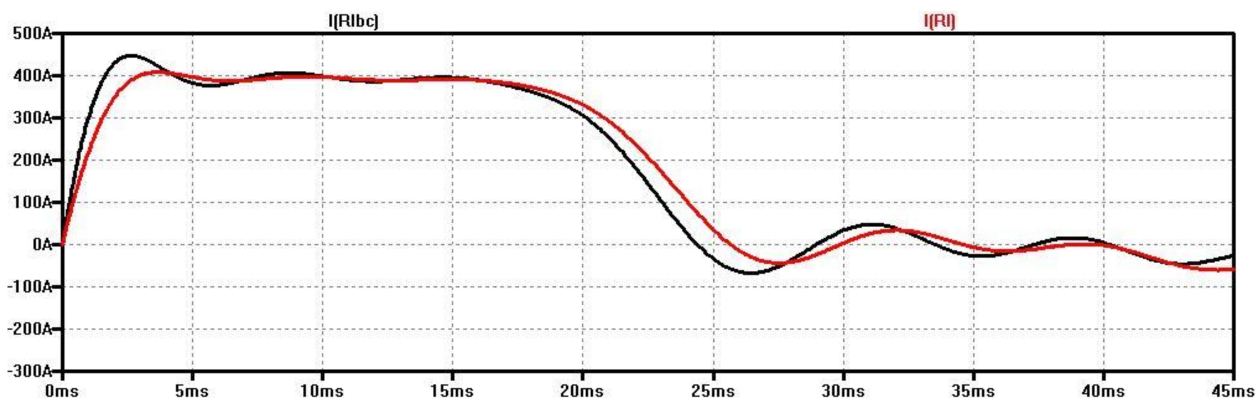
Arguably, the principal characteristic of a real wire-wound air-core inductor, other than its inductance, is the copper resistance, or Ohmic resistance, of the wires from which it is wound. Assuming that the coils are wound using heavy copper wire, the resistance of coils in the range envisioned will be small fractions of an Ohm. In fact, the resistance of the hookup wires which connect the inductors to the rest of the circuit may be greater than the Ohmic resistance of the coils. In any event, the following graph shows what happens when the inductors are given resistance of 0.02Ω.



The same phenomenon is observed here as it was when the capacitors had internal resistance. The internal resistance burns off energy on an on-going basis, so the current available to the load falls further and further below what it was in the resistance-free case. In the curve shown, the inductors had Ohmic resistance of $20\ \text{m}\Omega$, being one-fifth of the load resistance. Whether the coils need to be wound from heavier wire, so they have less resistance, depends on whether one can accept the smoothed-out curve shown.

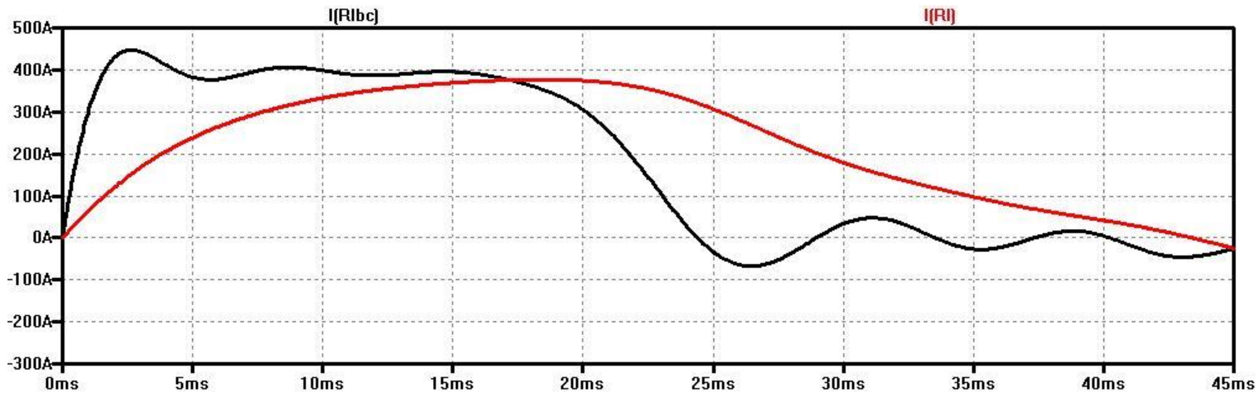
<u>Sensitivity #7 – Adding 100 mH and 1000 mH inductance to the load</u>

In many applications, the load of the pulse forming network will have an inductive component. Let us begin this sensitivity by examining the $R_L = 0.1\Omega$ load in series with an inductance of 100 mH. This is five times the inductance of each inductor in the network. The following graph shows what happens.

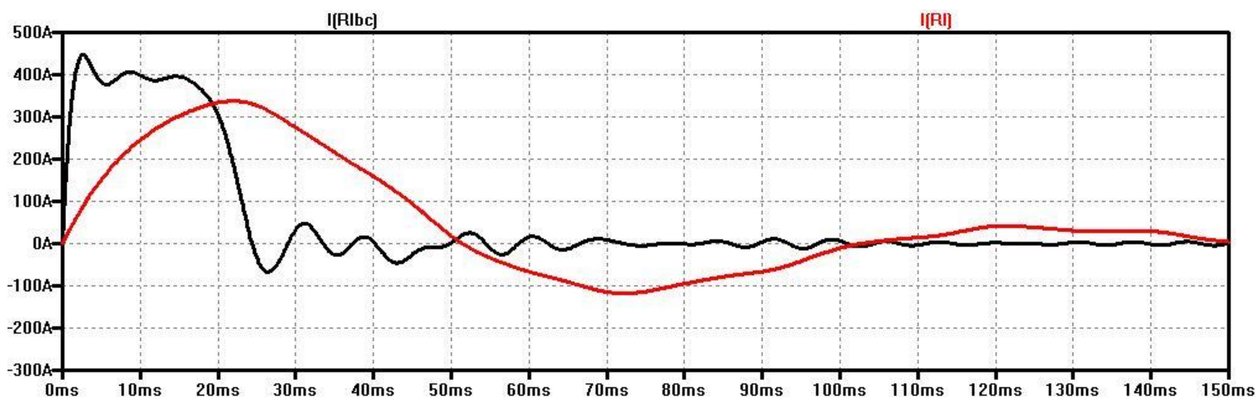


The effect on the pulse is modest and, generally, is a lengthening of the pulse.

In some applications, such as coil guns, the load is more inductive than resistive. The following graph shows the output current when the load is an inductance of 1000 mH in series with 0.1Ω



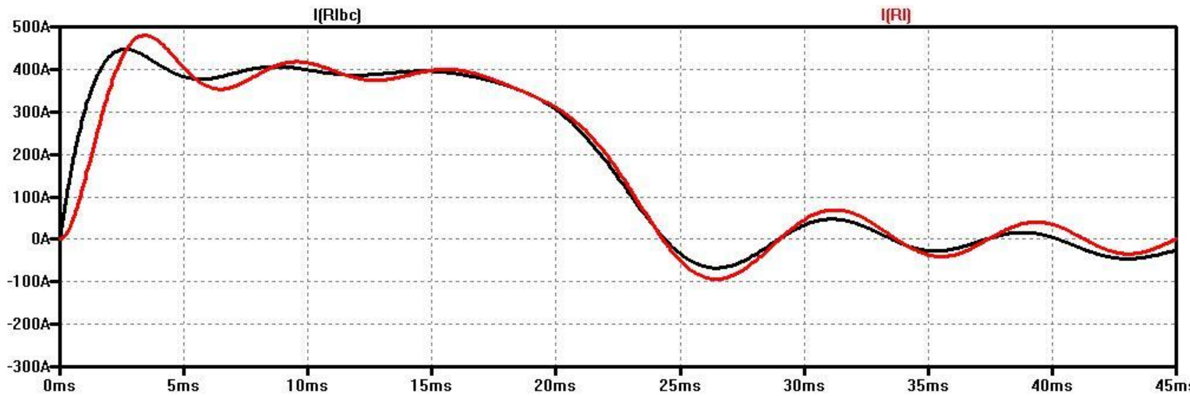
When the load is substantially an inductor, as it is here, the effect on the output current is very interesting. It looks more like the first pulse of a sine wave than a square wave. It has approximately the same peak magnitude and approximately twice the duration of the original pulse. Of course, these conclusions depend on the ratio of the load inductance to the network inductances. Furthermore, the output current will start to "ring" through the load inductance. The following graph shows a more extreme example, where the load inductance has been increased to $0.002 \text{ H} = 2000 \text{ mH}$, which is ten times the inductance per stage. The oscillations are more pronounced.



Of course, a condition where the load has an inductance of 2 mH is probably not unexpected. The designer always has the option of using the inductance of the load as all or part of the first-stage inductance L_1. If the load inductance is known, the choice of the capacitances and the inductances for the network could be made with that in mind.

<u>Sensitivity #8 – Adding 0.01 F capacitance to the load</u>

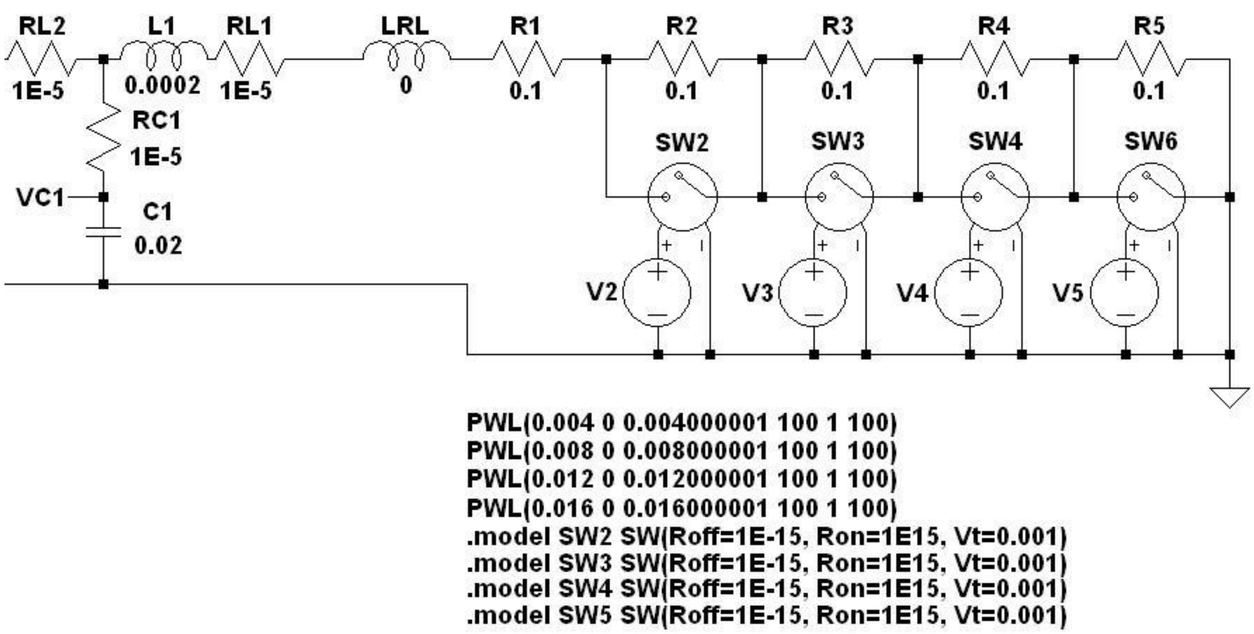
In this variation, we will add a 0.01 F capacitor in parallel with the 0.1Ω load resistor. This is a large capacitance, much larger than would turn up unexpectedly. The following graph shows (in red) the load current with the capacitance. The effect is surprisingly modest for so large a capacitor.



Sensitivity #9 – Where the load resistor increases with time

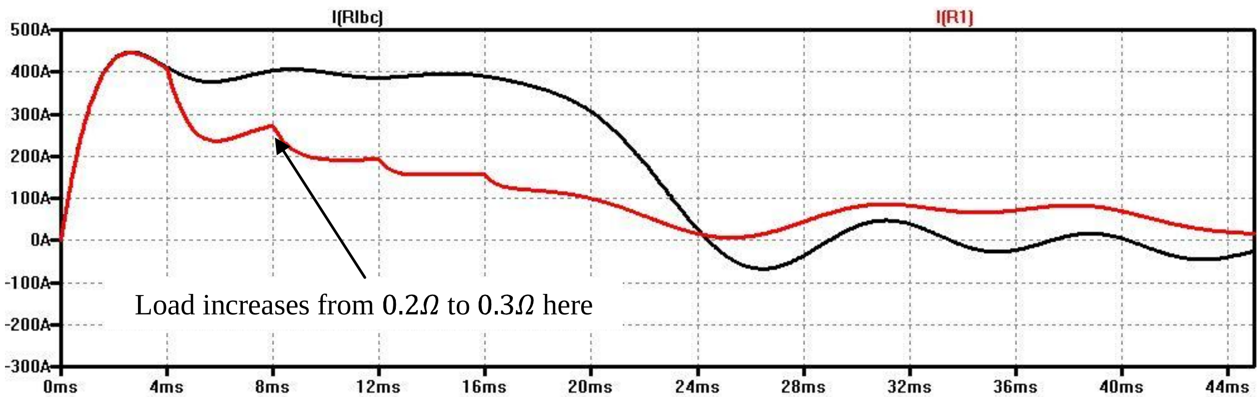
Now, let us look at a situation where the load resistance is not constant, but increases with time. The following figure is the SPICE schematic I used to simulate this condition. The pulse forming network is on the left side of the figure, but on the first stage is shown. (The resistors R_{L_1} and R_{L_2} represent the Ohmic resistance of the winding of the inductors in the first and second stages. The equivalent series resistance of the first stage's capacitor is resistor R_{C_1}. All three are set to zero. L_{R_L} is the inductance sometimes added in series with the load resistor. It, too, is set to zero for Sensitivity #9.)

The original 0.1Ω load is resistor R_1. As time progresses through the simulation, resistors R_2 through R_5 are added successively in series to R_1. The time at which the additional resistors are added to the load is determined by voltage-controlled switches SW_2 through SW_6. For example, voltage source V_2 goes high 4 ms into the simulation. The switches are normally closed, so switch SW_2 opens at that instant, placing resistor R_2 in series with R_1, but leaving resistors R_3 through R_5 shorted out. Resistors R_3, R_4 and R_5 are added at 8 ms, 12 ms and 16 ms, respectively.



Taken together, the load is a step resistance: 0.1Ω for the first 4 ms, 0.2Ω for the second 4 ms, and so on, rising to 0.5Ω after 16 ms, where it remains.

The following graph shows in red the current flowing through the step resistance. The usual base case, with the constant 0.1Ω resistor, is shown in black. A step increase in the load resistance causes a spike in the current, as there is an immediate increase in the drain of energy.



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An e-mail setting out errors and omissions would be appreciated.