An Axial Coil Gun

Addendum 1 – The current-carrying capacity of copper wire

Reference is made to the paper titled *The physics of an axial coil gun*. The coils used in that paper were wound using #4 gauge enameled copper wire. This is a big wire. Its diameter is 5.189 mm, which is equal to one-fifth of an inch. It has a correspondingly low resistance, of 0.2485 Ω per thousand feet, which is equal to 0.0008153 Ω per meter.

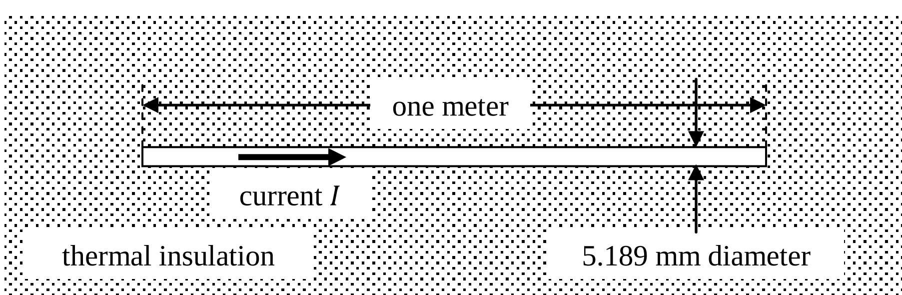
In the latter sections of that paper, I limited the current flowing through the coil to 100 Amperes. I imposed this restriction because of two properties set out in the standard American Wire Gauge table. For #4 gauge wire, the table specifies "Amperes" = 55.653 and "Maximum Amperes" = 83.480. The footnotes to the table describe these quantities as follows:

> "Amperes" is defined as the "conservative ampere rating based on 750 circular mils per ampere". [A "circular mil" is the area of a circle circumscribed by a square whose sides are 0.001 inch long. I have looked at other web sites which give similar figures, in the range of 700 to 750, for the area in circular mils required per ampere.]
>
> "Maximum Amperes" is defined as the "maximum allowable current based on 500 circular mils per ampere. Do NOT exceed this rating".

When I read the definition of "Maximum Amperes", and its "Do NOT exceed" warning, I believed that it was based on the extreme physical limits of copper, and was quite close to the current at which the electrostatic repulsive forces among the electrons in the material would cause the wire to explode. I have subsequently come to understand that this definition is not a physical limit *per se*, but a rule of thumb developed by utility companies for the safe and reliable transmission of power. While a coil gun should be safe and reliable, of course, it is not subject to the same conditions of use as power transmission lines. In this paper, we will look at some of the factors which gave rise to the utility company's rule of thumb and, by examining those factors, determine what rule we should impose on the wire used in a coil gun.

Thought experiment #1

Consider a one-meter length of #4 gauge wire carrying a steady current I. The wire is surrounded by thermal insulation which prevents any of the heat generated from dissipating. The situation is shown in the following diagram.



The total resistance of this piece of wire is $R = 0.0008153\Omega$. Because it is carrying a steady current, the power used to overcome the resistance, which is converted into heat, is $P = RI^2 = 0.0008153I^2$ Watts. Every second, additional energy (in the form of heat) is released into the wire. The heat added per second is given by $E_{per\ second} = 0.0008153I^2$ Joules.

Because the thermal insulation prevents any heat from escaping from the wire, the heat will remain inside the copper and its temperature will rise. The thermal capacity of a material such as copper is a measurable quantity that measures the amount of heat required to change the temperature of a given mass of the material by a given amount. This quantity is given by a substance's specific heat. For copper, the specific heat is $C_{copper} = 0.385$ Joules/gramKelvin, or 385 Joules/kgKelvin. In other words, 385 Joules of heat will raise the temperature of one kilogram of copper by one degree Kelvin. (One Kelvin degree is the same temperature difference as one Celsius degree.)

We next need to calculate the mass of the one-meter piece of wire. Its cross-sectional area (πr^2) is equal to $0.000021147\ \text{m}^2$, so its volume is equal to $0.000021147\ \text{m}^3$. The mass density of copper is $\rho_{copper} = 8.96\ \text{g/cm}^3$, or $8960\ \text{kg/m}^3$, so the piece of wire has a mass of $mass = 0.1895$ kg.

Since 385 Joules of heat will raise the temperature of one kilogram of copper by one degree Celsius, $0.0008153I^2$ Joules will raise the temperature of one kilogram of copper by $0.0008153I^2/385$, or $0.000002118I^2$, degrees Celsius. But, our piece of wire weighs less than one kilogram, so it will heat up faster. Our piece will heat up by $0.000002118I^2/0.1895$, or $0.00001118I^2$, degrees. Furthermore, this increase in temperature will continue for each second during which the current flows.

If we set the current to the AWG standard, of 83.480 Amperes, the increase in temperature will be $0.00001118 \times 83.480 \times 83.480 = 0.07791$ degrees Celsius every second.

If the piece of wire is enclosed in thermal insulation, the heat will accumulate and the temperature will keep rising. After 100 seconds, the temperature will have increased by 7.791 degrees Celsius from its initial temperature. For some purposes, such as a coil gun, this would be considered to be a very modest increase in temperature. 100 seconds of the "DO NOT exceed" current will heat the wire by 7.791 degrees Celsius – but that is vastly different from causing the wire to explode.

After 13,660 seconds, or 3 hours and 48 minutes, the temperature will have increased by $0.07791 \times 13{,}660 = 1064.25$ degrees. If the initial temperature was 20°C, then the temperature of the piece will have risen to 1084°C. This is the "melting point" of copper – the temperature at which the metal melts. (The resistance of the copper changes a bit as the temperature rises, but does not change the point. The copper will eventually melt.)

<u>Thought experiment #2</u>

Now, let us remove the thermal insulation. This will allow heat to escape from the wire, which will offset the increase in the temperature which the Ohmic resistance of the wire causes. Heat can escape from the wire by three different mechanisms: radiation, conduction and convection. As the temperature of the wire increases, an increasing amount of heat will be lost through these three mechanisms. There will be a temperature at which the rate at which heat escapes is equal to the rate at which heat is added. This temperature will be a point of equilibrium, after which the current will continue to flow but the temperature of the wire will remain constant.

In this particular thought experiment, we will examine only the rate at which heat escapes from the wire by means of radiation. Radiation in this instance means the emission of electromagnetic waves from the surface of the wire, in the infrared band of frequencies, somewhat below the frequencies of visible light. In order to model the situation, we will image heat radiating from the surface of the cylinder of the piece of wire (but not from the ends of the piece, which are, one assumes, connected to similar pieces) of wire.

The rate at which heat radiates from any surface is very highly dependent on the mechanical nature of the surface. A highly polished copper surface will radiate much less heat (at the same temperature) than a matte copper surface. If the surface is painted with a very thin layer of black paint, it will radiate more heat than if the surface is oxidized. And, so on. The rate at which heat is radiated from a surface is described by the Stefan-Boltzmann Law as follows:

$$\frac{Power}{Area} = \epsilon\sigma(T_S^4 - T_A^4) \quad (1)$$

where $Power$ is the energy radiated through area $Area$ per unit time. T_S is the temperature of the surface, measured in degrees Kelvin. (The units of temperature are important – although the temperature difference of one Kelvin degree is the same as the temperature difference of one Celsius degree, their zero-bases are different. Kelvin degrees are measured from absolute zero temperature, when all microscopic motion ceases. The melting point of pure water, which defines $0°\text{C}$, is equal to $273.15°\text{K}$.) In Equation (1), T_A is the temperature of the medium which surrounds the wire. Standard room temperature is often taken to be $20°\text{C}$, or $293.15°\text{K}$, but outdoor power transmission lines can experience ambient temperatures ranging from $-50°\text{C}$ in northern Canada to $+50°\text{C}$ in the desert.

Note that the radiation becomes more effective as the difference between the surface temperature T_S and the ambient temperature T_A increases. And, the effectiveness is highly non-linear – it is proportional to the difference between the fourth powers of the two temperatures.

In Equation (1), σ is a fundamental constant of nature called the Stefan-Boltzmann constant. It is written as:

$$\begin{aligned}\sigma &= \frac{2\pi^5 k^4}{15h^3c^2} \\ &= 5.670373 \times 10^{-8} \frac{\text{Joules}}{\text{m}^2\text{secondKelvin}^4} \quad (2)\end{aligned}$$

where k is the Boltzmann constant, h is Planck's constant and c is the speed of light. While the derivation of this constant is beyond the scope of this paper, it should be noted that it is derived by considering a "black body", which is an object which is a perfect absorber and emitter of radiation. For a black body, the factor ϵ in Equation (1) is equal to one. For real surfaces, the factor ϵ, which is called the emissivity, measures how well or poorly the surface radiates heat, when compared with a black body. Some values for copper, which depend on the surface treatment, are the following.

Surface treatment	**Emissivity ϵ**
Highly polished	0.02
Polished	0.03
Roughly polished	0.07
Matte	0.22
Rough	0.74
Black, oxidized	0.78
Cuprous oxide	0.87

For a power transmission line exposed to the weather, the surface can probably be considered as "rough" and may be "oxidized". A conservative assumption for the emissivity (where "conservative" in this context means less effective in radiating heat) would be $\epsilon = 0.75$.

Let us ask the main question for this thought experiment in the following way: if $0.0008153I^2$ Joules of heat are added to the piece of wire every second (this is $E_{per\ second}$ from above), and if radiation is the only means by which heat escapes from the wire, and if the emissivity of the surface is 0.75, and if the ambient temperature is $20°\mathrm{C}$, then what is the equilibrium temperature at which the wire will stabilize?

At equilibrium, the power leaving the wire must be equal to the power being added to the wire: $0.0008153I^2$ Watts. This is the $Power$ to be used in Equation (1). The area through which radiation occurs is the area of the curved surface of the cylinder, which will be the circumference $(2\pi r)$ of a cross-section of the wire, multiplied by its length. The surface is of our piece is $2\pi \times 0.002595 \times 1 = 0.1630\ \mathrm{m}^2$. With these parameters, we can write Equation (1) with the surface temperature as the only unknown:

$$
\begin{aligned}
\frac{Power}{Area} &= \epsilon\sigma(T_S^4 - T_A^4) \\
\rightarrow \quad \frac{0.0008153I^2}{0.1630} &= 0.75 \times 5.670373 \times 10^{-8}[T_S^4 - (20 + 273.15)^4] \\
\rightarrow \quad 117{,}600I^2 &= T_S^4 - 7{,}385{,}200{,}000 \\
\rightarrow \quad T_s &= \sqrt[4]{117{,}600I^2 + 7{,}385{,}200{,}000}\ \text{Kelvin} \qquad (3)
\end{aligned}
$$

If the current is $I = 83.480$ Amperes, then the surface temperature will be 300.97 Kelvin, or 27.8°C. The temperature of the wire will stabilize about eight degrees Celsius above the ambient temperature, which would probably be an acceptable temperature for the power companies. (The other mechanisms of heat loss, like convection into the surrounding air, will reduce the equilibrium temperature a bit.)

If the current is 1000 Amperes, the surface temperature will be 594.59 Kelvin, or 321.4°C. Rainwater would boil off the surface, which would not be acceptable to the power companies.

In neither case, though, is the copper anywhere close to melting.

Thought experiment #3

Now, let us consider the same wire again, but this time wound into the coil of a coil gun. The current flowing in the coil is not direct current. Far from it. It is a pulse. Not only is the pulse very short, just a few milliseconds, but the current during the pulse is not constant. The pulses we looked at in the earlier paper have more the shape of a half-wave of a sinusoid than a square wave. What this means is that the average heat generated inside the wire during the pulse is less than the maximum instantaneous heat generated when the current is at its maximum.

In this particular thought experiment, by way of an example, we will assume that the current pulse is a square wave with current I flowing for a duration of five milliseconds. (If the slug has not left the muzzle within five milliseconds, then it is not travelling very quickly.) If the coil gun is fired once per second, heat will be added to the wire for five milliseconds, but the wire would be able to cool down through radiation during the second which follows. The average power which must be dissipated from the wire is about $1/200^{\text{th}}$ of what it would be with steady current. And, since the temperature differences in the Stefan-Boltzmann Law are of the fourth order, the temperature rise would be much, much less than calculated in Thought experiment #2. We can repeat Equation (3), using a current of 1000 Amperes for example, as follows:

$$\frac{Power}{Area} = \epsilon\sigma(T_S^4 - T_A^4)$$

$$\rightarrow \frac{0.0008153 \times 1000 \times 1000/200}{0.1630} = 0.75 \times 5.670373 \times 10^{-8}[T_S^4 - (20 + 273.15)^4]$$

$$\rightarrow 588{,}070{,}000 = T_S^4 - 7{,}385{,}200{,}000$$

$$\rightarrow T_s = \sqrt[4]{7{,}973{,}300{,}000} \text{ Kelvin}$$

$$\rightarrow T_s = 25.7°C \tag{4}$$

Even with $1000\ Amperes$, the rise in temperature over room temperature is less than six degrees Celsius. Temperature rises in this range are simply not an issue for the coil gun application.

<u>The fusing current</u>

When looking at the literature, I see references to a current called the "fusing current". It seems that the fusing current is always described by reference to a square wave of current, where the duration of the pulse is specified. The shorter the pulse, the higher the current required to fuse, or melt, the wire.

I have not found any reference to a current which would instantly vapourize the wire.

Conclusion

The "Maximum Currents" set out in the standard American Wire Gauge table do not apply to coil guns. The short duration of the pulses in a coil gun permit the use of currents which are tens or hundreds of times greater than the "Maximum Currents" listed, before the resulting temperature rise becomes a concern.

<u>Re-focusing on our objective</u>

I have now concluded that the size of the wire we use in the coil will not be limited by considerations of temperature.

We do not want the coil in our coil gun to vaporize, or to melt, or even to come anywhere close to melting. On the other hand, we do want to use wire with the smaller diameter possible, since smaller diameters permit more turns and therefore greater applied magnetic fields. In theory, there will always be some minimum diameter at which heating becomes a concern. But, what is the minimum size of wire from a practical point-of-view?

The total resistance and inductance of the coil are also affected by the size of the wire, as well as by the number of turns and dimensions of the coil. The total resistance and inductance of the coil, together with the magnetic field it generates, impose their own restrictions on the wire which can be used. I believe that the other factors will impose the more binding restriction on wire size, and that the size of wire these other factors permit will be found to be acceptable from a thermal point-of-view.

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An e-mail setting out errors and omissions would be appreciated.