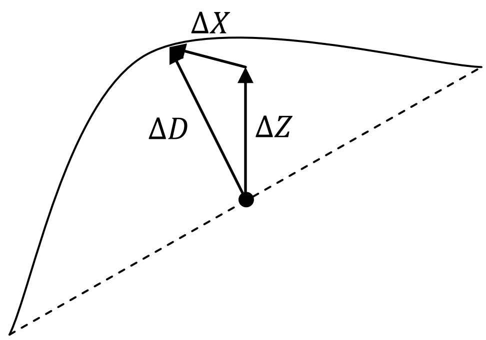
Version #4

Adding a trailing edge wire

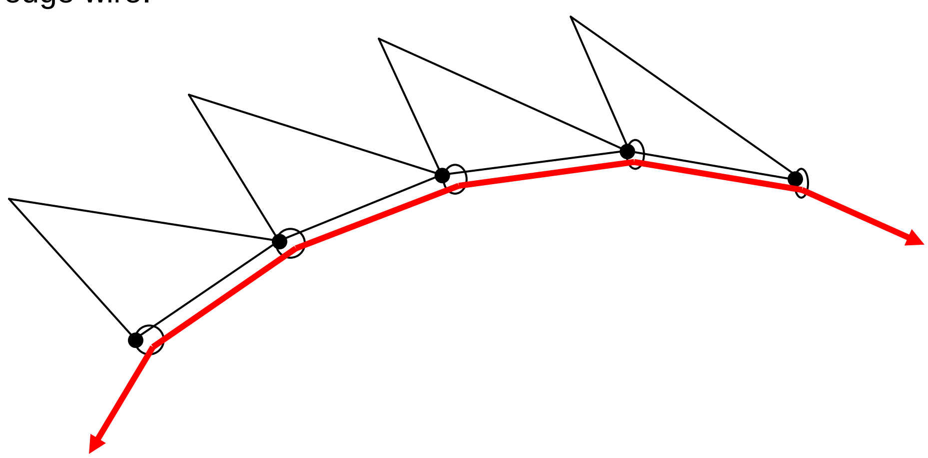
Consider the mid-point of the trailing edge of the kite in Version #3. Under the influence of the lift, this point moved[1] up in the Z-direction by 0.172 meters (ΔZ) and towards the nose by 0.122 meters (ΔX). That's a total displacement of 0.211 meters (ΔD).



The length of the trailing edge before any displacement was 12 feet, or 3.658 meters. After the final iteration, it was 3.688 meters (calculated by adding up the length of the line segments between all points on the trailing edge). This is an extension, or stretch, of the trailing edge by (3.688 - 3.658) / 3.658 = 0.82%.

My goal for this version is to find out if a trailing edge wire can limit the displacement of the center point on the trailing edge to, say, one-half of its displacement with fabric alone. In other words, I want ΔD to be, say, 0.1 meters.

It's usual to sew a hem along the trailing edge of the fabric and to run a small-diameter wire through the space created by the hem. I will use the following physical model for the trailing edge wire.



[1] The locations of points(100, 50) and (100,51) after the final iteration were set out in the SavedLocations text file as:

```
100 50 3.04545452562481 -0.018613597304234 0.171813301612158
100 51 3.04545452562481 0.018613597304234 0.171813301612158
```

The black dots are consecutive points along the fabric at the trailing edge. The triangles represent some of the grid triangles which meet at these points. I imagine that there is a small round ring sewn to the fabric at each of these points. The trailing edge wire is the red line passing through the rings. The arrows on the ends represent the pull of the tension acting along the axis of the wire.

The little rings are intended to be frictionless. They do not exert any force along the axis of the wire. In effect, they are like little pulleys, and allow the wire to slide through until it reaches an equilibrium position.

The main characteristic of this model is that the tension along the axis of the wire is the same at every point. Since the little rings cannot exert any axial force on the wire, there is nothing to hold the wire in place "sideways" except the wing tips of the kite, which are fixed in position (for now).

Now, let's take this model to the next step, and make the little rings vanishingly small. This means that the points of interest along the wire are exactly the same as the points already defined along the trailing edge of the fabric.

It follows that each point on the trailing edge actually has two ways of exerting a force on its neighbours on the trailing edge, through the fabric (as before) and through the tension in the wire. The two forces act in the same direction. We can even assume that the forces are additive.

I am going to treat the trailing edge wire like a spring, in much the same way as I treated stretching of the fabric. For the wire, though, we can calculate Hooke's constant quite readily.

Let's assume that the wire has a diameter of one millimeter. Its cross-sectional area πr^2 is 7.85 x 10^{-7} square meters. Young's modulus (elastic modulus, say E) for steel wire is said to be 183.9 GPa, or 1.839 x 10^{11} Newtons per square meter. If a piece of this wire has an unstretched length L_0 and is stretched by distance ΔL by some force F, then the stress divided by the strain is:

$$E = \frac{stress}{strain} = \frac{F \times L_0}{Area \times \Delta L}$$

$$\rightarrow \qquad F = E \times Area \times \frac{\Delta L}{L_0}$$

$$\rightarrow \qquad F = 1.839 \times 10^{11} \times 7.85 \times 10^{-7} \times \frac{\Delta L}{L_0}$$

$$\rightarrow \qquad F = 144{,}000 \frac{\Delta L}{L_0}$$

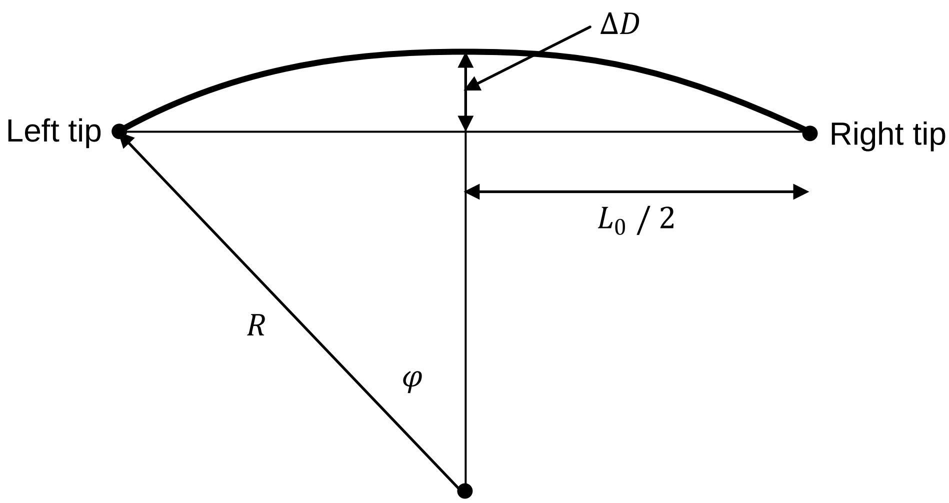
So, Hooke's constant for the wire is 144,000 Newtons. (The units are a little different than Hooke's constant for the fabric, because this constant is to be multiplied by the <u>fractional</u> elongation of the wire rather than its <u>absolute</u> elongation in meters.)

We now have two Hooke's constants. In the code, I use parameter names FabricHooke (previously named simply Hooke) and TEWireHooke to keep them separate.

What should be the unstretched length of the trailing edge wire?

When laying out the fabric in Version #1, I assumed that it was pulled flat between the leading edge tubes, but was not placed under any tension. I will make a different assumption for the wire. I will assume that the wire is a little bit longer than the fabric at the trailing edge. Therefore, the wire will not come into tension until the fabric has stretched a little bit.

To estimate how much slack should be allowed, let's assume that the trailing edge has the shape of a circular arc (it doesn't, but it's close). I set the target displacement at the center of the trailing edge above, as ΔD = 0.1 meters. The unstretched trailing edge has a length (see above) of L_0 = 3.658 meters. The circle which generates this arc is shown in the following figure.



The radius R of the generating circle (each row in the grid of points will have its own radius R) can be derived from the Pythagorean Theorem:

$$R^2 = (R - \Delta D)^2 + \left(\frac{L_0}{2}\right)^2$$

$$\rightarrow \quad 0 = -2R\Delta D + \Delta D^2 + \left(\frac{L_0}{2}\right)^2$$

$$\rightarrow \quad R = \frac{\Delta D^2 + \left(\frac{L_0}{2}\right)^2}{2\Delta D}$$

and angle φ can be found from:

$$\sin\varphi = \frac{L_0}{2R}$$

The length of the arc between the wing tips can be written down by inspection as a ratio of the circumference:

$$\frac{L}{2\pi R} = \frac{2\varphi}{2\pi}$$
$$\rightarrow \quad L = 2\varphi R$$

Substituting the target value for ΔD, we get:

$$R = \frac{(0.1)^2 + \left(\frac{3.658}{2}\right)^2}{2 \times 0.1} = 16.776 \text{ meters}$$
$$\varphi = \sin^{-1}\left(\frac{3.658}{2 * 16.776}\right) = 0.10924 \text{ radians}$$
$$L = 2 \times 0.10924 \times 16.776 = 3.6653 \text{ meters}$$
$$\rightarrow \quad L - L_o = 3.6653 - 3.658 = 0.0073 \text{ meters}$$

I have kept five decimal places because the displacement is extremely sensitive to the elongation. To get our desired displacement of 10 centimeters at the mid-point of the trailing edge, the wire needs to be only 7 millimeters or so longer than the unstretched trailing edge.

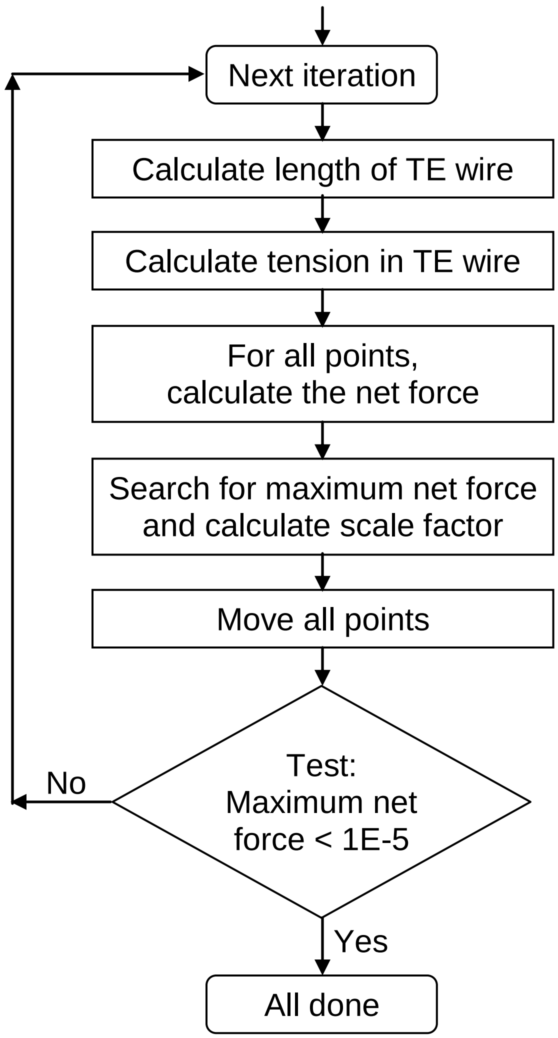
<u>What changes to the code are needed to handle the trailing edge wire?</u>

Answer: very few.

The procedure to home in on the equilibrium shape of the fabric which was developed in Version #3 makes no assumptions about the shape. It simply moves points in the grid in the direction of the net force acting on them.

The only points which are directly affected by the trailing edge wire are, of course, the points on the trailing edge itself. The existing procedure does not need any changes to handle points in the "interior" of the fabric. Even for those points on the trailing edge, the effect is limited to their two neighbours which are also located on the trailing edge. The directions to the neighbours can be calculated as before. The only change needed for point-to-point interactions along the trailing edge is to add the tension force due to the wire to the tension force due to stretching of the fabric. The relevant block of code from the Procedures.vb module is this:

```
' Calculate the fabric tension (scalar) exerted by this neighbour
lTension =
    FabricHooke * (lDistance - Point(Irow, Icol).RestDistanceToNeigh(Ineigh))
' Limit the fabric tension to positive values only
lTension = Math.Max(0, lTension)
' For points and neighbours on the TE, add the tension in the TE wire
If ((Irow = NumRows) And (lRowNeigh = NumRows)) Then
    lTension = lTension + TEWireTension
End If
```



Changes to the main procedure, in module BuildKite.vb, are summarized in the flow chart to the left. The length and tension of the trailing edge wire is calculated at the start of every iteration. Then, as before, the net forces acting on all grid points is calculated. The point subjected to the greatest net force is allowed to move a preset distance (stored in variable MaxAllowedMove), and a scale factor is used to move all other points a distance in proportion to the net forces acting on them.

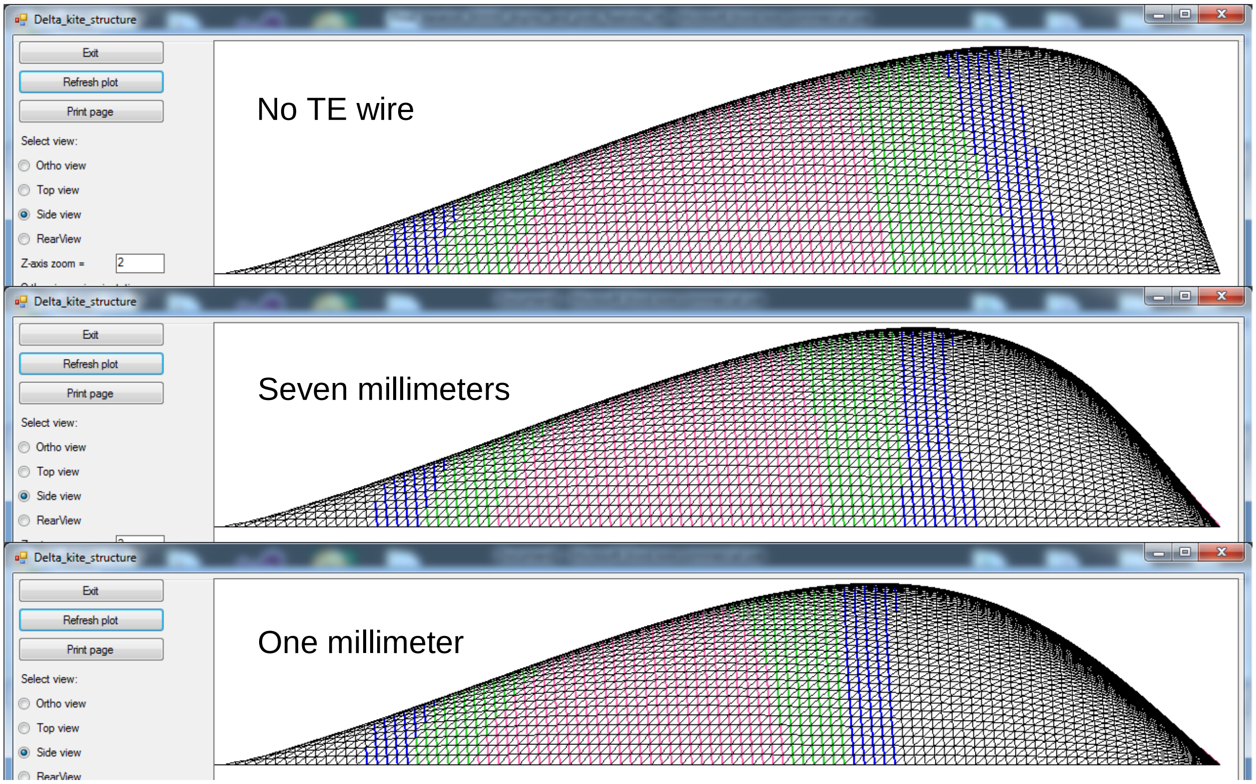
As before, the maximum allowed distance of movement increases or decreases depending on whether or not the net forces in successive iterations are decreasing or not.

Simulations results

The following figure is a composite of three plots showing the side view of the kite. The first shows the shape with no trailing edge wire. The second shows the shape when the trailing edge wire has a "slack" of seven millimeters. The third is the shape with a slack of only one millimeter.

As expected, the surface does become flatter as the trailing edge is increasingly pulled back. Even so, all of these shapes are much too "baggy" to be of use in generating aerodynamic lift. The rear part is so convex that the airflow will separate from the surface.

With one millimeter of slack, the wire holds the trailing edge almost straight across from wingtip to wingtip, but it is still not enough to pull the fabric flat. The remedy is to cut the fabric so the trailing edge is concave, not straight. I will do this in a later version.

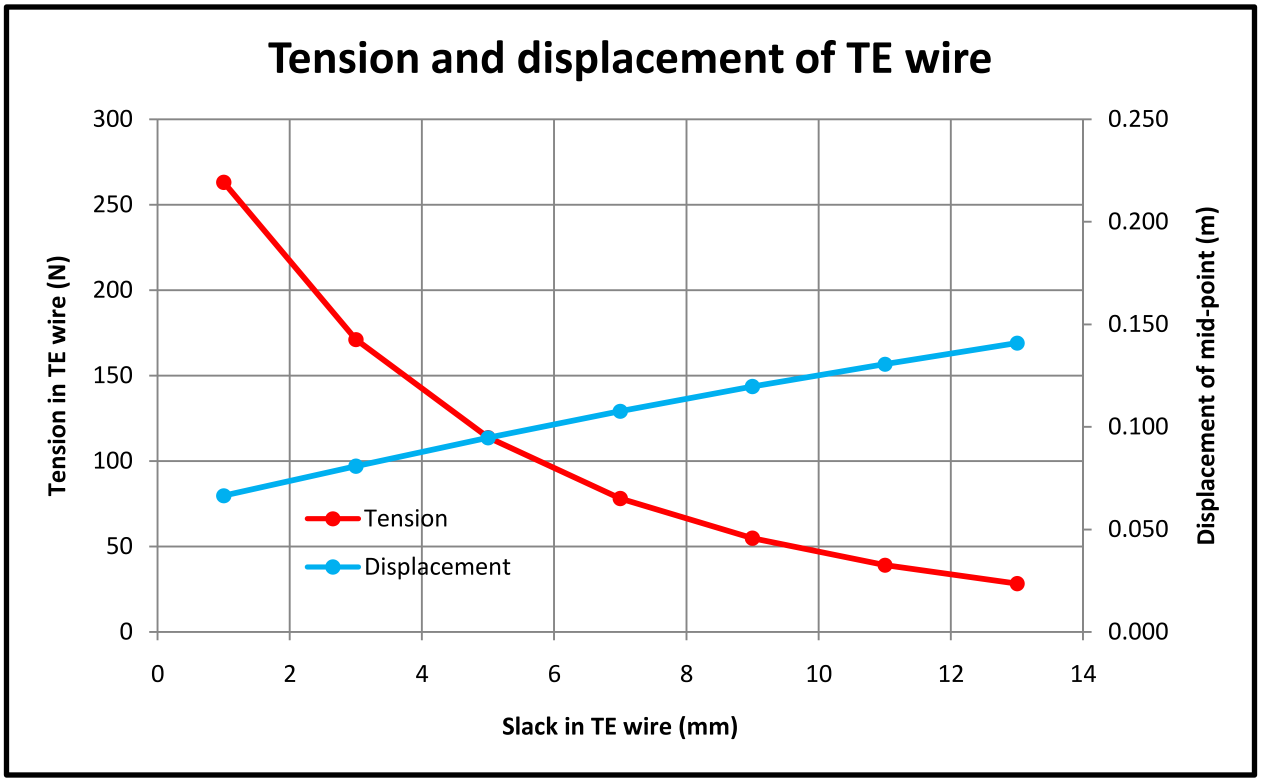


<u>The magnitude of the tension in the trailing edge wire</u>

Using a trailing edge wire is not without cost, the "cost" being the force which it exerts on the ends of the leading edges. Unfortunately, the leading edges will not be made of steel I-beams -- whatever tubes or rods they are made of will bend inwards at the wingtips. The tubes or rods will have to be stout enough to withstand the tension in the wire. And that means weight.

The following graph shows the tension force in the wire (in red, with respect to the left-hand vertical axis) and the ΔD displacement of the mid-point of the trailing edge (in blue, with respect to the right-side vertical axis). The horizontal axis is the slack build into the wire, in millimeters.

For example, leaving a slack of only one millimeter causes a tension in the wire of 270 Newtons, which is almost 70 pounds. That is a huge amount of force to exert, pulling sideways no less, at the end of a tube which is 12 feet long (the length of the leading edges in our default kite).



In the next version, I will start to analyze the structure of the leading edges. I will assume they are round carbon fiber tubes, and see how they bend in response to the forces exerted by the fabric and the trailing edge wire.

Jim Hawley
August 2021

(As always, an e-mail describing errors and omissions would be appreciated.)